ISSN 1063-7710, Acoustical Physics, 2009, Vol. 55, No. 6, pp. 698–707. © Pleiades Publishing, Ltd., 2009.

CLASSICAL PROBLEMS OF LINEAR ACOUSTICS AND WAVE THEORY

On the Low-Frequency Sound Scattering in a Moving Microinhomogeneous Medium

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Received December 15, 2008

Abstract—The Rayleigh law that governs low-frequency sound attenuation due to the scattering by inhomogeneities in a microinhomogeneous medium is generalized to the case of particles moving in a flow or falling under gravity. Corrections to the scattering's cross section that adjust the Rayleigh law to the case of a potential flow around inhomogeneities are calculated. It is shown that, when microinhomogeneities are moving in a viscous medium, the characteristics of discrete scatterers may considerably deviate from the Rayleigh law. Based on the data on the velocity and size distribution of falling drops of water in air, refinements are proposed for the laws of low-frequency sound scattering by rain.

PACS numbers: 43.20.Fn, 43.28.Fp, 43.28.Gq

DOI: 10.1134/S1063771009060025

The phenomenon of wave scattering in an inhomogeneous medium lies at the basis of our notion of the surrounding world. Although its basic concepts were formulated by Tyndall and Rayleigh as far back as in the 19th century, it still attracts the attention of researchers [1–5]. For example, in solving the reference problem of sound scattering by a fixed sphere in a viscous fluid [6], it was found that neglect of the viscous term in the Curle equation may lead to erroneous estimates of the dipole components of the scattered field. Requirements concerning the strictness of the analysis of such problems are explained by the crucial effect of wave scattering by the simplest inhomogeneities on adequate estimates of many physical phenomena, in particular, low-frequency sound scattering in a microinhomogeneous medium. This phenomenon is well known and seems to be rather simple at first glance. For example, in a forest, one can see through very short distances, because, for the small wavelength of light, the scattering's cross section of a leaf is large and is determined by the double area of the leaf. By contrast, sounds in a forest can be heard through large distances, because, according to the Rayleigh law, for the long sound waves, the scattering's cross section makes an infinitesimal fraction of the leaf area. Analogously, in fog or rain, one can clearly see objects only at arm's length while the sounds can be heard at large distances, which are almost identical to those observed in fair weather.

These examples illustrate the scattering of sound by numerous small bodies. Evidently, the frequency spectrum of the field scattered by fixed bodies is the same as that for a single scatterer. In can be shown that, in the case of chaotically positioned scatterers, the total scattered power is identical to the power scattered by a single body multiplied by the total number of bodies [1, 5].

Microinhomogeneous media represent sets of scatterers spaced at distances that are small compared to the wavelength. At the same time, the minimal distance between the particles considerably exceeds the particle size. If identical inhomogeneities were uniformly distributed in a medium with a homogeneous concentration, e.g., in the form of a periodic lattice, no scattering would be observed and only a slight change in the sound propagation's velocity would occur [1]. As is known, in this case, the side spectra of a small-scale lattice represent rapidly attenuating inhomogeneous waves. According to the optical analogy, in a regular crystal, light waves scattered by individual molecules cancel each other everywhere except for the direction of the initial wave.

However, we are interested in the chaotic distribution of obstacles with their concentration being constant only on the average. The theory of low-frequency sound scattering in such media is based on the laws that govern the scattering of sound by an individual inhomogeneity whose size is small compared to the wavelength: $ka \ll 1$ (k is the wave number and a is the characteristic size of the particle). For inhomogeneities at rest, the classical Rayleigh law is valid [1, 2, 6, and 7]. According to this law, the scattering's cross section σ of an inhomogeneity is proportional to the cross-sectional area of the body πa^2 multiplied by the small quantity $(ka)^4$.

A microinhomogeneous medium can be characterized by the concentration of scatterers n and the specific scattering's cross section $n\sigma$, which determines the scattering property of a unit volume of the scattering medium. The wave intensity decreases due to the scattering by inhomogeneities as an exponential function of distance x: $W = W_0 e^{-n\sigma x}$. The logarithmic attenuation γ characterizing the sound intensity's decay with distance in terms of decibels per unit length of the sound's propagation path has the form $\gamma = 4.3n\sigma$. For the mean radius a of inhomogeneities and their mean concentration n, which is expressed through the volume of an inhomogeneity and the total volume's fraction τ occupied by the material of inhomogeneities in the medium as $n = 3\tau/4\pi a^3$, the quantity γ is determined by the formula $\gamma \cong 1.04\tau\sigma/a^3$. For example, if we apply the Rayleigh law in classical form to drops of water with a radius of 1 mm and assume that the drops are chaotically distributed in air so that the water content in air is 5% ($\tau = 0.05$), and the attenuation γ will be $\gamma \cong 1.31 \cdot 10^2 (ka)^4$ per meter or $\gamma \cong 1.31 \cdot 10^5 (ka)^4$ per kilometer. The attenuation rapidly decreases with decreasing frequency. In the frequency band within 100–1000 Hz, which determines the speech intelligibility, γ is as small as $1.87 \cdot 10^{-6}$ to $1.87 \cdot 10^{-2}$ dB/km.

At the same time, it is evident that most of the elements of natural microinhomogeneous media are in motion. For example, the leaves of trees are moved by wind and the droplets of fog or rain move under the effect of gravity. Sometimes, the motion in the medium is of an oscillatory character, such as in the case of leaves moved by wind [5], and, sometimes, it represents a uniform gravity drop, such as in the case of fog or rain droplets [8]. For radio or light waves, the motion-related corrections to the scattering's cross sections are negligibly small and can be ignored (the corrections are proportional to the ratio of the velocity of scatterer motion to the velocity of wave propagation). However, for sound waves, such corrections, being proportional to the hydrodynamic Mach number, are considerable and should be taken into account. In addition, the idea that the frequency spectrum of the field scattered by many bodies is identical to that produced by a single scatterer fails in the case of moving bodies. According to the Doppler effect, the spectrum of the scattered field depends on the angle of the wave incidence on the direction of the scatterer motion. Below, it will be shown that the effect of inhomogeneity motion on the scattered field is not only determined by the Doppler effect: it is of both a kinematic and dynamic nature.

The purpose of this study is generalization of the Rayleigh law to the case of moving elements of a microinhomogeneous medium and, in particular, to the case of a uniform motion of microinhomogeneities, e.g., carried by a flow or falling under gravity. The results are illustrated by estimates of sound attenuation due to the scattering by falling drops of rain or fog.

The distribution of rain drops in size and velocity has been studied over more than 150 years. In [8], it was noted that small drops of water freely falling in air acquire a spherical shape under the effect of capillary forces. As the drop diameter varies from 0.1 to 4–5 mm, the velocity of the falling drops increases with the drop size from 0.3 to 10 m/s. For drops whose diameter is smaller than 0.1 mm, the velocity rapidly decreases; specifically, when the drop diameter is 0.01 mm (10 μm), the velocity decreases down to 3×10^{-5} m/s. For drops with a diameter greater than 4–5 mm, the concept of the spherical shape fails. Under the effect of the gradually increasing pressure difference, a drop is deformed: it flattens primarily in the region of the critical point and sometimes even acquires a recess. When the drop diameter reaches a value of about 6.5 mm, the drop decays. The behavior of the drop velocity V (m/s) depending on the drop diameter d (mm) is determined by the laws governing the variation of hydrodynamic drag forces with varying Reynolds number Re [8]. Initially, for d increasing up to 0.1 mm, the Stokes law $V \cong 31d^2$ is obeyed. In the transition region, for d from 0.1 to 1 mm, $V \approx 4d$. Finally, in the region where regular drag forces take place, for d from 1 to 4–5 mm, $V = 4.6\sqrt{d}$.

Thus, on the whole, the initial data for fog or small rain are as follows: the velocity of drops is up to 0.3 m/s while the diameter of drops is up to 0.1 mm. For common rain, the velocity of drops is within 4–10 m/s, and the drop diameter is within 1–5 mm. The water content in air in the case of fog or rain is denoted by w and expressed in grams of water per cubic meter or per kilogram of dry air (g/m^3 or g/kg, respectively). The actual values of water content are within several units to several tens of units [3]. Evidently, for water drops in air, $\tau = 1.2 \times 10^{-6} w$. In nature, the drops of fog or rain are distributed in size within the limits indicated above. As a rule, the distributions used to estimate the attenuation are of an empirical character [2, 3].

The flow around the moving inhomogeneities (e.g., falling water drops) is close to the potential one in the major part of the surrounding space [9–13]. Therefore, the effect of such a flow on scattering is of primary interest. Let us study this phenomenon by considering the flow formed near a spherical inhomogeneity that moves with a constant velocity V in an ideal liquid. Unlike the approach used in [6], we describe the sound propagation near the inhomogeneity by the Lighthill equation, as in [9–13]. For a monochromatic wave with a frequency ω, this equation has the form

$$\Delta p + k^2 p = \frac{2i}{\omega}\frac{\partial}{\partial x_\alpha}\left(U_\beta \frac{\partial^2 p}{\partial x_\alpha \partial x_\beta}\right), \quad (1)$$

where $k = \omega/c$ and p is the acoustic pressure.

To complete the formulation of the problem, i.e., to write the control equation with all the appropriate additional conditions, it is necessary to determine the relation between the acoustic pressure p and the scalar potential φ determining the velocity of liquid particles in the sound wave: $\mathbf{V} = \nabla\varphi$. Using the Euler equation of motion with allowance for the liquid flow, in the linear approximation in M, we obtain that the desired relation in the moving coordinate system $\mathbf{r}' = \mathbf{r} - \mathbf{V}t$ is as follows:

$$p(\mathbf{r}', t) = i\omega\rho\left\{\varphi(\mathbf{r}', t) + \frac{i}{\omega}[U_\alpha - V_\alpha]\frac{\partial\varphi}{\partial x'_\alpha}\right\}. \quad (2)$$

In view of relation (2) between the variables φ and p, a mathematical problem can be formulated for the potential φ, as well as for the acoustic pressure p. However, we apply another approach, which was used in [11, 12].

Let us formulate the solution to Eq. (1) as in [9, 10], namely, for the calibration potential Ψ. This potential is related to the scalar velocity's potential in the sound wave, $\mathbf{V} = \nabla\varphi$, by the formula $\Psi(\mathbf{r}) = \varphi(\mathbf{r})\exp(-ik\mathbf{r}\mathbf{M})$. Here, the renormalized wave number k is expressed through the Doppler frequency $\omega = \omega_0(1 - \mathbf{M}\mathbf{n}_0)$, where $\mathbf{n}_0$ is the unit vector in the propagation direction of the incident plane's monochromatic wave and $M = V/c$ is the hydrodynamic's Mach number. For the field component Ψ_{sf}, which describes the sound scattering by the inhomogeneities of the velocity $U(r)$ characterizing the accompanying flow of the medium, the equation represented in the moving coordinate system $\mathbf{r}' = \mathbf{r} - \mathbf{V}t$ has the form [10–12]

$$\Delta'\Psi_{sf} + k^2\Psi_{sf} = -\frac{2ik}{c}\left[U_\alpha\frac{\partial\Psi}{\partial x'_\alpha} + n_{0\alpha}n_{0\beta}\frac{\partial U_\alpha}{\partial x'_\beta}\Psi_i^{(0)}\right]. \quad (3)$$

Here, $\Psi = \Psi_i + \Psi_{sp} + \Psi_{sf}$ is the total wave's field satisfying the Lighthill equation, $\Psi_i^{(0)}$ is the incident plane's monochromatic wave in the zero-order approximation in the hydrodynamic's Mach number, and Ψ_{sp} is the calibration potential corresponding to the wave reflected from the moving surface of the body and satisfying the homogeneous Helmholtz equation. In the case of a potential flow of an ideal liquid around a sphere, the velocity distribution $\mathbf{U}(\mathbf{r})$ in the medium is described by the formula

$$\mathbf{U}(\mathbf{r}') = \frac{a^3}{2r'^3}[3(\mathbf{V}\mathbf{n})\mathbf{n} - \mathbf{V}], \quad (4)$$

where a is the radius of the sphere and $\mathbf{n} = \mathbf{r}'/r'$ is the unit vector directed from the sphere's center $\mathbf{r}_0(t) = \mathbf{V}t$ to the observation point $\mathbf{r} = \mathbf{r}' + \mathbf{V}t$. For simplicity, in what follows, the primes indicating the spatial coordinates in the moving coordinate system will be omitted.

Unlike [10, 11], the sound scattering by a potential flow will be considered below for the general case, where the dimensionless parameter ka takes an arbitrary value. In addition, we will consider the meaning of the solution Ψ_{sf} and its consequences. In the linear approximation in M, the solution to Eq. (3) can be represented in the form

$$\Psi_{sf} = \frac{ik\Psi_0}{2\pi c}\int d^3r_1 G(\mathbf{r}, \mathbf{r}_1)(\mathbf{n}_0\nabla)(\mathbf{n}_0\mathbf{U}e^{ik\mathbf{n}_0\mathbf{r}_1}) + \frac{ik}{2\pi c}\int d^3r_1 G(\mathbf{r}, \mathbf{r}_1)(\mathbf{U}\nabla)\Psi_{sp}^{(0)}. \quad (5)$$

Here, $\Psi_0 = p_0/\rho c$ is the amplitude of the incident field and $\Psi_{sp}^{(0)}$ is the field scattered by the surface of the sphere at rest (the reflected wave in the zero-order approximation in M). The integrals in Eq. (5) over the region outside the sphere can be calculated by taking the function $G(\mathbf{r}, \mathbf{r}_1)$ to be the Green function of free space: $G(\mathbf{R}) = R^{-1}\exp(ikR)$.

It is known that, when low-frequency sound is scattered by a stationary sphere whose radius is small compared to the wavelength of sound, the fraction of scattered waves is very small and the scattering amplitude is proportional to k^2a^3 [1, 7, and 10]. Then, from the comparison of two terms in Eq. (5), it follows that the second term of the solution can be ignored, as it was done in [10, 11]. However, to obtain the solution for the general case, i.e., to determine Ψ under the condition that the parameter ka is arbitrary, the second term in Eq. (5) should be taken into account [12].

Note that, although the solution to Eq. (3) in the Born approximation is represented in analytical form (5), it is ambiguous. In [9, 10], it was shown that, for moving bodies, the separation of the total scattered field Ψ_s into two components, one of which, Ψ_{sp}, is related to sound scattering by the body and the other, Ψ_{sf}, to sound scattering by the flow, is only conditional and has no actual physical meaning. Such a separation is convenient for computations and for analyzing the physical features of the general solution. The conditionality of separating the scattered field into individual components follows from the fact that the uniqueness of the solutions to the respective equations for Ψ_{sp} and Ψ_{sf} requires setting individual additional conditions at the boundary $r = a$. However, at the surface of the body, only one boundary condition for the total field is set, which, for a perfectly rigid inhomogeneity, has the form [10]

$$\left(\frac{\partial\Psi}{\partial r}\right)_{r=a} = -ik\mathbf{n}\mathbf{M}\Psi(a). \quad (6)$$

Since the total field Ψ is formed as the sum $\Psi_i + \Psi_{sp} + \Psi_{sf}$, in which two terms represent independent unknowns, the separation of Eq. (5) into two individual conditions for the fields Ψ_{sp} and Ψ_{sf} can be done in many different ways. In connection with this, in [9, 10], it was proposed that a specific solution to Eq. (3) can be found by using its partial solution, which, being represented in the form of Eq. (5), is expressed through the Green function for free space. Then, condition (6) serves for an unambiguous determination of the unknown scattered field Ψ_{sp} under the assumption that the forms of the functions Ψ_i and Ψ_{sf} are known. The additional problem of finding the field Ψ_{sp} scattered by the surface of the moving sphere and satisfying the Helmholtz equation is actually reduced to the common problem of sound scattering by a body. However, together with the initial incident plane wave Ψ_i, an additional field Ψ_{sf} is now scattered by the body. By choosing another Green function in Eq. (5), it is possible to find another partial solution to Eq. (3). This results in a change in the specific form of the boundary condition for determination of the scattered wave Ψ_{sp}, and, hence, the specific form of the solution Ψ_{sp} will change. Still, the newly determined solution $\Psi = \Psi_i + \Psi_{sp} + \Psi_{sf}$ should in total satisfy both the initial equation for the total field Ψ and the boundary condition given by Eq. (6). Therefore, by virtue of the uniqueness of the solution to the problem, the expressions that were obtained for the total field in different ways should coincide.

Thus, the field component (5) determined with the use of the Green function for free space represents a certain fictitious imaginary field, which would occur in the liquid in the presence of the flow (4) without the body. Since the second term in Eq. (5) describes the scattering by the flow for the wave reflected from the body while the wave in question always allows expansion in plane waves, let us first consider the scattering by the inhomogeneities of the flow velocity for a plane's monochromatic wave. This situation is described by the first term of Eq. (5). As was shown in [9–11], the corresponding fictitious field can approximate the actual field Ψ_{sf} scattered by the flow inhomogeneities in a number of cases (e.g., when $ka \ll 1$). From the expression $\Psi_{sf} \cong \Psi_0 F_f \exp(ikr)/r$, which is valid for $r \longrightarrow \infty$, we determine the scattering amplitude $F_f(\mathbf{n}, \mathbf{n}_0)$ for the plane wave Ψ_i scattered by the inhomogeneities of the flow (4) surrounding the moving sphere. The scattering amplitude has the form [10]

$$F_f = \frac{ikn_{0\alpha}n_{0\beta}}{2\pi c}\int d^3r_1 e^{-ik\mathbf{n}\mathbf{r}_1}\frac{\partial}{\partial x_{1\alpha}}[U_\beta(r_1)e^{ik\mathbf{n}\mathbf{r}_1}], \qquad (7)$$

where integration is performed over the entire region $r_1 \geq a$ occupied by the flow.

From Eq. (4) for the potential flow's velocity $\mathbf{U}(\mathbf{r})$ and a rough estimate of integral (7), it follows that the region adjacent to the sphere surface makes a considerable contribution to integral (7). Therefore, extension of the region of integration in Eq. (7) to the entire space, including the region $0 \leq r' \leq a$ (as it is done in some of the publications) may lead to an error. Let us perform a more accurate integration in Eq. (7) and determine the scattering amplitude F_f for arbitrary values of the parameter ka. For this purpose, we first take the integral by parts by representing it in the form

$$F_f = -\frac{k^2(\mathbf{n}\mathbf{n}_0)}{2\pi c}\int\limits_{r_1 \geq a} d^3r_1(\mathbf{U}\mathbf{n}_0)e^{i\mathbf{q}\mathbf{r}_1} - \frac{ik}{2\pi c}\int\limits_{r_1 = a}(d\mathbf{S}_1\mathbf{n}_0)(\mathbf{U}\mathbf{n}_0)e^{i\mathbf{q}\mathbf{r}_1}. \qquad (8)$$

Here, the volume integral with the total divergence of the integrand is transformed to the surface integral according to the Gauss theorem. In this case, the integral taken over the surface S_∞ lying infinitely far from the body vanishes, because the velocity of the liquid $\mathbf{U}(\mathbf{r})$ decreases with distance from the sphere center as r^{-3} according to Eq. (4) while the area of the surface S_∞ increases as r^2. The wave vector $\mathbf{q}$, which has the meaning of the "momentum" transferred to the medium, is $\mathbf{q} = k(\mathbf{n}_0 - \mathbf{n})$, and its magnitude is $2k\sin(\theta/2)$, where θ is the scattering angle determined from the equality $\cos\theta = \mathbf{n}\mathbf{n}_0$. Substituting the potential flow's velocity (4) in Eq. (8), we obtain a specific expression for the desired scattering amplitude F_f. Calculation of the integrals obtained in this way is rather complicated, and it is described in [12].

Using the results of calculating the volume and surface integrals, from Eq. (8) we obtain

$$F_f(\mathbf{n}, \mathbf{n}_0) = \frac{k^2a^3}{2}\left\{[(\mathbf{M}\mathbf{n}_0) - 3(\mathbf{M}\mathbf{n})]\frac{j_1(qa)}{(qa)} + 3[(\mathbf{M}\mathbf{n}_0)(1 + \mathbf{n}\mathbf{n}_0) + (\mathbf{M}\mathbf{n})(3 - 5\mathbf{n}\mathbf{n}_0)]\frac{j_2(qa)}{(qa)^2}\right\}, \qquad (9)$$

where $j_1(qr)$ and $j_2(qr)$ are first- and second-order Bessel's spherical functions. The expression obtained for F_f is valid for any value of ka. Using general expression (9), it is possible to determine the partial scattering amplitude that characterizes the low-frequency sound scattering by the liquid flow formed near an inhomogeneity with a small radius. Assuming that $ka \ll 1$, we expand the Bessel's spherical functions involved in Eq. (9) in powers of this small parameter. Then, we obtain the following formula for the scattering amplitude $F_f(\mathbf{n}, \mathbf{n}_0)$ in the case of $ka \ll 1$:

$$F_f = \frac{k^2a^3}{5}\left\{(\mathbf{M}\mathbf{n}_0)\left[\frac{4}{3} + \frac{1}{2}(\mathbf{n}\mathbf{n}_0)\right] - (\mathbf{M}\mathbf{n})\left[1 + \frac{5}{2}(\mathbf{n}\mathbf{n}_0)\right]\right\}. \qquad (10)$$

This formula coincides with the corresponding expression given for F_f in [10]. In [12], a rigorous calculation of the scattering amplitude F_p was performed for scattering by a moving inhomogeneity. In the limit $ka \ll 1$, the expression obtained in [12] for an arbitrary value of ka takes the following form accurate to the terms on the order of M^2:

$$F_p = \frac{(k^2a^3)}{5}\left\{-\left[\frac{5}{3}+\frac{1}{2}(\mathbf{Mn}_0)\right]\right.$$
$$+\frac{1}{2}[5(\mathbf{nn}_0)-(\mathbf{nn}_0)(\mathbf{Mn}_0)-3(\mathbf{Mn})] \quad (11)$$
$$\left.+\frac{5}{2}\left[(\mathbf{Mn})(\mathbf{nn}_0)-\frac{1}{3}(\mathbf{Mn}_0)\right]\right\}.$$

Combining expressions (10) and (11), we obtain the total scattering amplitude for sound scattering by a moving body:

$$F = k^2a^3\left[-\frac{1}{3}+\frac{1}{2}\mathbf{n}(\mathbf{n}_0-\mathbf{M})\right].$$

From Eqs. (10) and (11), it follows that the correction to the scattering amplitude in the potential flow formed around a moving scatterer is proportional to k^2a^3M. The correction is anisotropic, because the expansion in spherical harmonics contains monopole, dipole, and quadrupole components. Taking the squared magnitude of amplitude (10) and integrating it with respect to angles, we determine the partial scattering's cross section σ_f for sound scattering by potential flow (4). Calculations show [10, 12] that this quantity is expressed as

$$\sigma_f = \frac{3}{25}\pi k^4a^6M^2\left(1+\frac{5}{27}\cos^2\theta_0\right); \quad ka \ll 1, \quad (12)$$

where θ_0 is the angle between the vector $\mathbf{n}_0$ and the velocity $\mathbf{V}$ of the body ($\cos\theta_0 = \mathbf{n}_0\mathbf{V}/V$).

From Eqs. (10) and (12), it follows that the partial scattering's cross sections characterizing the sound scattering by inhomogeneities of the potential flow near moving microinhomogeneities are proportional to the square of the hydrodynamic's Mach number. However, as it was mentioned above, sound is scattered not only by the flow of the medium that is caused by the motion of the body, but also by the moving surface of the sphere itself. When sound is scattered by fixed microinhomogeneities with a small radius ($ka \ll 1$), the scattering amplitude is proportional to k^2a^3 and practically isotropic, so that the scattering's cross section is proportional to k^4a^6 in compliance with the Rayleigh law [1, 7]. For an inhomogeneity with an arbitrary density and an arbitrary compressibility, under the condition that $ka \ll 1$, the scattering's cross section $\sigma^{(0)}$ has the form [1, 12]

$$\sigma^{(0)} = \frac{4}{9}\pi k_0^4a^6\left[\left(1-\frac{\rho c^2}{\bar{\rho}\bar{c}^2}\right)^2+3\left(\frac{\bar{\rho}-\rho}{2\bar{\rho}+\rho}\right)^2\right], \quad (13)$$

where c and $\bar{c}$ are the velocities of sound in the liquid and in the sphere, respectively; ρ and $\bar{\rho}$ are their densities; and $k_0 = \omega_0/c$ is the wave number. The motion of the sphere with a velocity $V \ll c$ gives rise to corrections to the amplitude $F^{(0)}$ and the scattering's cross section $\sigma^{(0)}$ due to the sound scattering by both the flow and the moving surface of the body. As shown above, the partial scattering's amplitude associated with the flow F_f is proportional to k^2a^3M irrespective of the sphere radius. As for the diffraction corrections $F_p^{(1)}$ associated with the sound scattering by the moving surface of the body, they have the structure of Eq. (11) similar to that of expression (10) for the scattering amplitude associated with the flow. For a small-radius sphere, this result was obtained in [10–12].

Thus, calculation of the full scattering's cross section for sound scattering by a moving inhomogeneity with allowance for both the wave diffraction by its moving surface and the wave scattering by inhomogeneities of the accompanying flow of the surrounding liquid leads to the appearance of additional terms in expressions of the type of Eqs. (10) and (11). In addition to the term $\sigma^{(0)}$, which is zero-order in the Mach number and is described by Eq. (13), and the terms that are quadratic in M and arise due to the linear corrections to the amplitude F in the expression for the cross section σ, cross terms arise being proportional to the Mach number. Thus, the total scattering's cross section for sound scattering by a small-radius particle moving with the velocity $V \ll c$ and surrounded by potential flow (4) is [10, 12]

$$\sigma = \frac{7}{9}\pi k_0^4a^6(1-6\mathbf{Mn}_0). \quad (14)$$

This formula expresses the modified Rayleigh law for incompressible inhomogeneities uniformly moving in a potential flow. For a sphere with arbitrary density and compressibility, the modified Rayleigh law in the case of $ka \ll 1$ takes the form $\sigma = \sigma^{(0)}(1-6\mathbf{Mn}_0)$, where $\sigma^{(0)}$ is given by Eq. (13).

Under the assumption that the flow around the drops is the potential, the application of the modified Rayleigh law to drops of rain that have a diameter of 5 mm and fall uniformly with a velocity of 10 m/s at a water content in air of 5% ($\tau = 0.05$), yields the attenuation $\gamma \cong 9.3(ka)^4(1-0.2\cos\theta_0)$ per meter or $\gamma \cong 9.3 \cdot 10^3(ka)^4(1-0.2\cos\theta_0)$ per kilometer. For example, in the frequency band 100–1000 Hz (which determines the speech intelligibility), the attenuation of sound propagating through rain in the horizontal direction ($\cos\theta_0 = 0$) is very small, as could be predicted earlier, and, according to the classical Rayleigh law, is only 5×10^{-6} to 5×10^{-2} dB/km. However, in the case of sound propagation at other angles to the flow, the attenuation due to sound scattering by rain varies. For example, in the direction opposite to the drop motion, i.e., upwards ($\cos\theta_0 = -1$), the attenuation increases by 20%, whereas, in the downward direction ($\cos\theta_0 = 1$), the attenuation decreases by 20%. A further increase in the velocity of microinhomogeneities could basically lead to an increase in anisotropy, but, at such velocities, the flow around microinhomogeneities, such as drops, cannot be purely potential [8]. Still, it should be noted that, in a microinhomogeneous medium, at relatively small velocities of microinhomogeneities on the order of 50–60 m/s ($M \cong 0.16$), the scattering is doubled in the case of upstream sound propagation in air and virtually absent in the case of downstream sound propagation.

Let us consider the frequency dependence of sound scattered by the inhomogeneities of the accompanying flow of liquid. It is similar to the frequency dependence of sound scattered by the moving body itself, because it is determined by the same time factor multiplying the scattering amplitudes F_p and F_f. If, in the factors $\exp(ik|\mathbf{r} - \mathbf{V}t| - i\omega t)$, we expand the quantity $|\mathbf{r} - \mathbf{V}t|$ in the small parameter $V(t - t_0)/|\mathbf{r} - \mathbf{r}_0|$, we find that the time dependence is determined by the ordinary exponential time factor: $\exp(i\omega_s t)$. The frequency of the scattered field depends on both the angle of wave incidence and the angle of wave observation and has the form

$$\omega_s = \omega_0(1 - \mathbf{M}\mathbf{n}_0 + \mathbf{M}\mathbf{n}). \qquad (15)$$

Formula (15) is derived under the assumption that, in the moving coordinate system, the Doppler frequency is $\omega = \omega_0(1 - \mathbf{M}\mathbf{n}_0)$. From Eq. (15) it follows that, at a stationary spatial point r, the frequency of scattered sound ω_s varies as a function of the observation angle and may coincide with the incident wave's frequency ω_0 in two cases. First, this may occur when sound is scattered at a zero angle, i.e., when $\mathbf{n} = \mathbf{n}_0$. Second, the frequencies may coincide when sound is scattered at an arbitrary angle under the condition that the velocity vector $\mathbf{V}$ is perpendicular to the difference between the unit vectors $\mathbf{n}$ and $\mathbf{n}_0$. In particular, if the scattering region is observed in the transmission geometry, the frequency shift $\omega_s - \omega_0$ will be absent at the instant the body crosses the transmitter–receiver path irrespective of the angle of crossing.

To make estimates for the case of sound scattering by inhomogeneities of a viscous liquid flow caused by a small-size inhomogeneity [11], we assume, as above, that the velocity of the moving inhomogeneity V is constant and small compared to the sound velocity in the medium c. Then, the hydrodynamic Mach number $M = V/c$ satisfies the inequality $M \ll 1$. In addition, if the radius of the microinhomogeneity is sufficiently small, the corresponding Reynolds number $\mathrm{Re} = aV/\nu$ is also small, and the flow around the inhomogeneity will obey the Stokes law while the velocity distribution $\mathbf{U}(\mathbf{r})$ in the liquid will be described by the formula $\mathbf{U} = \mathrm{curl}(g\mathbf{V})$ [7, 11, and 14], where the function $g(r')$ in the coordinate system $\mathbf{r}' = \mathbf{r} + \mathbf{V}t$ satisfies both the equation

$$\mathrm{grad}\Delta'^2 g = 0, \quad r' > a \qquad (16)$$

and the boundary conditions that follow from the requirement that the velocity at the sphere surface, i.e., at $r' = a$, be zero and that $U' \longrightarrow -V$ at $r' \longrightarrow \infty$. The solution to Eq. (16) that is valid in the external region $r' > a$ and decreases at infinity can be found in the form $g = ar + b/r$. Using the boundary conditions to determine the unknown coefficients a and b, we obtain the velocity distribution $\mathbf{U}(\mathbf{r}')$ in the liquid:

$$\mathbf{U}(\mathbf{r}') = -\mathbf{V} - \frac{a^3}{3}\frac{3(\mathbf{V}\mathbf{n})\mathbf{n} - \mathbf{V}}{r'^3} + \frac{3a}{4}\frac{(\mathbf{V}\mathbf{n})\mathbf{n} + \mathbf{V}}{r'}, \qquad (17)$$

where $\mathbf{n} = (\mathbf{r} - \mathbf{V}t)/|\mathbf{r} - \mathbf{V}t|$ is the unit vector directed at the observation point $\mathbf{r}$. Remember that, in the moving coordinate system, the velocity $\mathbf{U}(\mathbf{r})$ is expressed as $\mathbf{U}(\mathbf{r}') + \mathbf{V}$.

As is known, Eq. (1) is correct to linear terms in the hydrodynamic's Mach number, but it initially ignores the viscosity and the variation in entropy of the liquid due to dissipation processes related to heat conduction and viscosity of the medium. The inclusion of dissipation in the zero-order approximation in the hydrodynamic's Mach number leads to attenuation of propagating waves. For example, for a plane's monochromatic wave of the form $p_0\exp(i\mathbf{k}\mathbf{r} - i\omega t)$, the inclusion of the rejected terms in Eq. (1) leads to a change in the wave number $k = \omega/c$, i.e., to the appearance of its imaginary part [15, 16]

$$\mathrm{Im}k = \frac{\omega^2}{2c^3}\left[\left(\frac{4}{3}\nu + \frac{\zeta}{\rho}\right) + \lambda(\gamma - 1)\right],$$

where ν and ζ/ρ are the viscosity coefficients, λ is the thermal diffusivity, and $\gamma = c_p/c_v$ is the specific heat ratio. The inclusion of these rejected terms actually leads to renormalization of the wave number k in Eq. (1), which is assumed to be accomplished in the subsequent calculations. An equation that is more general than Eq. (1) is the Blokhintsev–Howkins (sometimes called Ffowes–Williams–Howkins) equation [15, 16]. It also contains the cross terms that are linear in the Mach number M and proportional to the first power of dissipation coefficients. If these coefficients and the Mach number are small, the aforementioned additional terms remain small compared to the terms that are already present in Eq. (1) and, hence, can be ignored in the first approximation. Thus, the sound propagation in a viscous medium with allowance for the accompanying flow near a moving body can also be described by Eq. (1) by taking into account that, in this case, vorticity of the flow is nonzero while the velocity distribution near the body is described by, e.g., formula (17). Note that, in this case, the velocity can formally be represented as the sum of two terms $\mathbf{U}(\mathbf{r})$, in which $\mathbf{U}_1$ is described by the second term of Eq. (17) and $\mathbf{U}_2$ by the third term of Eq. (17). The first term of Eq. (17), i.e., $-\mathbf{V}$, is related to the shift of the coordinate system and is unimportant for the scattering problem.

Now, it should be noted that the expression for the velocity component $\mathbf{U}_1$ is similar in structure to Eq. (4) and differs from it only in the coefficient, which is $(-1/2)$ times smaller than that of the potential flow's velocity. Therefore, representing the total sound scattering's amplitude F_{fvisc} by the sum of two terms, $F_{\text{fvisc}} = F_1 + F_2$, each of which is determined by the respective component U_k, we immediately obtain the expression for the amplitude component F_1 determined by the flow $\mathbf{U}_1$. Using the result of [10], where the scattering amplitude was found for sound scattering by inhomogeneities of potential flow (4), and taking into account the aforementioned factor $(-1/2)$, we obtain that the amplitude component F_1 makes one half of Eq. (11). The scattering amplitude component F_2 is calculated on the basis of Eq. (7), in which the velocity $\mathbf{U}(\mathbf{r})$ is taken in the form $\mathbf{U}_2 = 3a[(\mathbf{V}\mathbf{n})\mathbf{n} + \mathbf{V}]/4r$. Unlike $\mathbf{U}_1$, this velocity component decreases slower with distance from the body (as $1/r$), which leads to an increase in the value of the integral and determines the vortex character of the viscous flow. Indeed, direct calculation shows that $\operatorname{curl}\mathbf{U}_1 = 0$ and the total vorticity of the flow $\mathbf{\Omega} = \operatorname{curl}\mathbf{U}$ is determined by the velocity component $\mathbf{U}_2$ and is expressed by the formula $\mathbf{\Omega} = 3a[\mathbf{V} \times \mathbf{n}]/2r^2$. After integration, we obtain that, when $ka \ll 1$, the scattering amplitude's component F_2 of sound scattered by the flow $\mathbf{U}_2$ is approximately given by the expression

$$F_2 \cong \frac{3}{4}a(\mathbf{M}\mathbf{n}_0 + \mathbf{M}\mathbf{n}). \tag{18}$$

From the comparison of Eqs. (11) and (18), it follows that the component F_2 of the sound scattering's amplitude associated with the scattering by the vortex's flow component is greater than F_1 by a factor of $(ka)^2$ and does not depend on frequency. Hence, as the frequency decreases, the ratio of these amplitudes rapidly increases. Since the total scattering amplitude $F_f = F_1 + F_2$ is in this case determined by the component F_2 while F_1 makes only half the scattering amplitude associated with the scattering of sound by inhomogeneities of the potential flow, we can conclude that, in the case of $ka \ll 1$, the inclusion of the viscosity of the liquid leads to a considerable increase in the sound scattering's amplitude.

However, from accurate calculation of the previously rejected part of integral (5), i.e., the part related to the velocity $\mathbf{U}(\mathbf{r})$ rather than its derivative (8), it follows that, in fact, this part is not small and should also be taken into account. Direct calculation of integral (5) with allowance for the second term in its integrand formally leads to divergence of the integral. This is a consequence of the slow decrease in velocity (17) with distance. It should be remembered that the Stokes-type velocity distribution in a viscous liquid (Eq. (17)) holds only in the region adjacent to the body, whereas, away from the body, the velocity decreases faster than $1/r$ [7, 8, and 14]. Hence, the region of integration in Eq. (5) can be physically limited to a distance on the order of a/Re, within which distribution (17) is valid. As a result, the scattering amplitude F_f proves to be finite. The estimate of integral (5), as the previously calculated value of expression (18) for the amplitude $F_f \approx F_2$, proves to be much greater than the scattering amplitude of sound scattered by inhomogeneities of the potential flow. The corresponding partial scattering's cross section considerably exceeds the value of Eq. (12) and, for $ka \ll 1$, is expressed as

$$\sigma_f = \frac{3}{4}(\pi a^2)M^2[3\cos^2\theta_0 + 1].$$

To calculate the full scattering's cross section, it is necessary to consider three cases depending on the relative values of M and $(ka)^2$.

Taking into account that $F_1 = F_{\text{fpot}}/2$, where F_{fpot} is given by Eq. (10) and the sum $F_{\text{pot}} = F_p + F_{\text{fpot}}$ is given by Eq. (11), we denote $F_2 = F_{\text{fvisc}}$ and obtain the total scattering's amplitude in the form $F_{\text{visc}} = F_p + F_f = F_{\text{pot}} + F_2 - F_1/2$. The squared magnitude $|F_{\text{visc}}|^2$ used for calculating the scattering cross section is

$$|F_{\text{visc}}|^2 \cong F_{\text{pot}}^2 + 2F_{\text{pot}}F_{\text{fvisc}} + F_{\text{fvisc}}^2. \tag{19}$$

This expression neglects the quantities $|F_{\text{fpot}}|^2$, $2|F_{\text{fpot}}||F_{\text{fvisc}}|$, and $2|F_{\text{pot}}||F_{\text{fpot}}|$, which are proportional to the product of the cross-sectional area of the inhomogeneity by $M^2(ka)^4$, $M^2(ka)^2$, and $M(ka)^4$, in comparison with the terms proportional to $(ka)^4$, $M(ka)^2$, and M^2, respectively.

If $M > (ka)^2$, we have $F_2 = F_{\text{fvisc}} > F_1$ and, calculating the scattering cross section $\sigma_{\text{visc}} = \int d\Omega |F_{\text{visc}}|^2$, it is possible to ignore not only the term proportional to F_1^2, but also the first term of Eq. (19), which is proportional to $(ka)^4$; thus, we retain only the second and last terms of Eq. (19), which are proportional to $M(ka)^2$ and M^2. The scattering cross section takes the form

$$\sigma_{\text{visc}}[M > (ka)^2] = \frac{1}{2}(\pi a^2)\left\{\frac{3}{2}[3(\mathbf{Mn}_0)^2 + M^2] - (ka)^2(\mathbf{Mn}_0)\right\}. \tag{20}$$

If $M < (ka)^2$, we have $F_2 = F_{\text{fvisc}} < F_1$. In this case, it is possible to retain only the first term of Eq. (19), which is proportional to $(ka)^4$. The scattering obeys the modified Rayleigh law for the case of a potential flow around the body, and the expression for the scattering cross section coincides with Eq. (14):

$$\sigma_{\text{visc}}[M < (ka)^2] = \frac{7}{9}(\pi a^2)(ka)^4(1 - 6\mathbf{Mn}_0).$$

Finally, if $M \approx (ka)^2$, we have $F_2 = F_{\text{fvisc}} \approx F_1$. Then, in Eq. (19), only the terms proportional to $M(ka)^4$ can be ignored while the three terms that are proportional to $(ka)^4$, $M(ka)^2$, and M^2 are retained. In the first term, it is possible to ignore the addition proportional to M in parentheses of Eq. (14). The expression for the scattering cross section takes the form

$$\sigma_{\text{visc}}[M \approx (ka)^2] = (\pi a^2)\left\{\frac{7}{9}(ka)^4 + \frac{3}{4}[3(\mathbf{Mn}_0)^2 + M^2] - \frac{1}{2}(ka)^2(\mathbf{Mn}_0)\right\}. \tag{21}$$

Applying the scattering law refined with allowance for the viscous nature of the flow around inhomogeneities, for water drops with a diameter of 0.1 mm that are falling uniformly with a velocity of 0.31 m/s in air with a water content of 5% ($\tau = 0.05$), the attenuation γ for $M > (ka)^2$ takes the following form according to Eq. (20): $\gamma \cong 3.37 \cdot 10^{-3}\left[\frac{3}{4} + \frac{9}{4}\cos^2\theta_0 - \frac{(ka)^2}{2M}\cos\theta_0\right]$ per meter or $\gamma \cong 3.37\left[\frac{3}{4} + \frac{9}{4}\cos^2\theta_0 - \frac{(ka)^2}{2M}\cos\theta_0\right]$ per kilometer. These estimates are obtained under the assumption that all the drops are identical; i.e., the distribution of drops in size is ignored [3]. In the frequency band 100–1000 Hz, the attenuation of sound propagating in such a medium is relatively strong even for horizontal propagation ($\cos\theta_0 = 0$): it exceeds the values predicted by the classical Rayleigh law by several orders of magnitude. In addition, the attenuation is almost frequency independent and, for the chosen conditions, makes about 2.24 dB/km. The contribution of the viscous flow to scattering in the horizontal direction proves to be determining. For sound propagating in the directions different from horizontal, attenuation somewhat increases and begins slightly depending on both the frequency and direction of wave propagation. Thus, along with the similarities, some distinctions from the case of a potential flow are observed. The sound attenuation for upward propagation, against the motion of drops ($\cos\theta_0 = -1$), is several times greater than that for downward propagation, in the direction of drop motion ($\cos\theta_0 = 1$), as in the case of the potential flow. However, for sound propagating downwards ($\cos\theta_0 \geq 0$), the attenuation at low frequencies exceeds that at high frequencies. Conversely, for sound propagating upwards ($\cos\theta_0 \leq 0$), the attenuation at high frequencies is greater than that at low frequencies. On the whole, the frequency dependence of attenuation is rather weak.

If the diameter of drops decreases by half, the velocity of their motion decreases fourfold. The scattering cross section of a drop decreases by a factor of 16 (if $M > (ka)^2$), while the attenuation, according to Eq. (20), decreases by a factor of 8. For example, for drops with a diameter of 0.05 mm, the attenuation still considerably exceeds the classical one determined by the Rayleigh law. It makes 0.28 dB/km, but now only for sound frequencies below 50 Hz; for drops with a diameter of 0.01 mm, this occurs for sound frequencies below 0.1 Hz. Thus, as the size of drops decreases, the relation $M > (ka)^2$ for sound is violated. Generally speaking, fog usually means air with water drops whose size is within 0.01 mm [2]. Therefore, drops with a size of 0.01 to 0.1 mm should be classified with drizzle rather than fog. Hence, the specific viscous scattering law given by Eq. (20) is only valid for low-frequency sound and water drops whose size lies within 0.1 to 0.01 mm. On further decrease in the drop size or on further increase in frequency, the value of the Mach number approaches $(ka)^2$ and, at $M \approx (ka)^2$, Eq. (21) begins to be valid. For $M < (ka)^2$, the scattering obeys the modified Rayleigh law (14). Evidently, for fog droplets with a diameter smaller than 0.01 mm, the fall of the droplets can be ignored. For drops of rain with a diameter greater than 0.1 mm, the scattering is mainly determined by the modified Rayleigh law (14) with allowance for a certain refinement concerning the role of the laminar wake, which is specified below.

Such considerations are also applicable to estimating the effect of a decrease in the volume fraction τ of water in air (the water content) on the predicted attenuation value. Evidently, the attenuation value should be proportional to the water content. To estimate the actual values of τ from below, we note that, in the case of precipitation, the water content w ($\tau = 1.2 \times 10^{-6}w$) exceeds (sometimes, by a factor of several tens) the values of the water content corresponding to saturation conditions. The latter depend on temperature and make 4 to 30 g/kg (grams of water per kilogram of dry air) in the temperature range from 0 to 30°C. With allowance for the mean value of w, which under saturation conditions is about 10 g/kg ($T \approx 20$°C), the viscous attenuation due to the scattering of waves for all the frequencies below 1000 Hz should be about 5.6×10^{-4} dB/km for rain drops with a diameter of 0.1 mm. This value is at least five orders of magnitude greater than the classical attenuation predicted by the Rayleigh law for a frequency of 1000 Hz under the same conditions and also considerably exceeds the attenuation calculated with allowance for the viscous and heat conduction losses [3, 4]. As the sound frequency decreases, the mechanism of the growth of the scattering's cross section due to the contribution of the viscous flow near the microinhomogeneity persists, which, along with the expected decrease in viscous absorption [1, 3], emphasizes the significance of the effect considered above. The theoretical values of attenuation of sound intensity in air at normal conditions are determined by the expression $\delta = 3.1 \times 10^{-9} f^2$ (in decibels per kilometer). However, the values of sound attenuation observed experimentally in air in the frequency band from 100 to 1000 Hz far exceed the theoretical values and make 0.5×10^{-3} to 0.15 dB/km. In [2], the authors present experimental data on sound attenuation in fog, which were obtained by measuring the sound attenuation in a reverberation chamber with drops whose mean size was 0.0065 mm. In [3, 4], one can find theoretical estimates of sound attenuation due to viscosity and heat conduction in a medium with drops (the viscosity manifests itself as the entrainment of suspended aerosol particles by the sound wave). Most of the estimates belong to the frequency range above 500 Hz, but, in the low-frequency part of this range (500–1000 Hz), the predicted attenuation values are much smaller than our estimates. Still, the attenuation due to the sound scattering in a medium with drops (at a drop diameter of 0.1 mm and a volume fraction of water $\tau = 5 \times 10^{-2}$) that was predicted with allowance for experimental data is not high. In particular, being an estimate from above, it requires no correction of the parameters of technical sound generation means (e.g., warning horns operating in fog or rain within distances of about 1–2 km). However, for sound waves with frequencies of 100, 10, or 1 Hz, such an attenuation value seems to be anomalous or at least unexpected. This statement holds even if one takes into account both the decrease in attenuation with a certain decrease in the size of drops and the actual value of water content for precipitation under consideration.

It should be noted that, for drops of rain with diameters of 1–5 mm and with velocities of downward motion on the order of several meters per second, such estimates of attenuation values are hardly valid. The actual influence of the viscous flow on the sound scattering by rain drops of this size cannot be estimated by the Stokes law, because the drag coefficient for falling rain drops, which determines the flow of the surrounding liquid, proves to be many times lower [8]. The above estimates of the effect of viscosity on the scattering are restricted by the limiting diameter of water drops, up to 0.1 mm (0.01–0.1 mm), and the limiting velocity of the viscous motion of drops, up to 0.3 m/s ($\mathrm{Re} \approx 2$). On further increase in the velocity of motion or in the size of the microinhomogeneity, with an increase in the Reynolds number, the flow around the inhomogeneity acquires the character of a laminar wake [8, 13].

Thus, one can see that, in the general case, the motion of an inhomogeneity with respect to the direction of wave propagation modifies the Rayleigh law in the whole range of angles of their intersection. In [13], it was shown that the laminar wake accompanying the actual motion of microinhomogeneities (drops of rain), being extended in the longitudinal direction, can additionally affect the scattering's cross section. This requires a further refinement of the modified Rayleigh law (14) for sound scattering by rain. However, in solving this problem, the calculation of the integrals in Eq. (8) for $ka \ll 1$ proves to be complicated, and such calculations require special consideration in another paper.

In closing, it should be noted that the scattering property of microinhomogeneities that was calculated above is somewhat overestimated, since it ignores the irreversible loss of mechanical energy in inhomogeneity oscillations, which is observed together with secondary sound radiation. This loss leads to a decrease in amplitude and, hence, to a decrease in scattering [1–5, 15, and 16].

On the other hand, the presence of viscosity affects the structure of the flow near the moving inhomogeneity. In particular, a potential flow develops vorticity and, in addition, the decay of the flow parameters with distance becomes slower. This leads to an increase in the scattering's cross section according to the law linear in the Mach number. Evidently, the second effect is noticeably greater than the first one, especially for ordinary, i.e., nonresonant inhomogeneities, such as falling drops of water.

Thus, the corrections to the scattering's cross section are calculated for the case of a potential flow around a moving inhomogeneity. These corrections, being proportional to the hydrodynamic's Mach number, modify the Rayleigh law of low-frequency sound attenuation in a microinhomogeneous medium (Eq. (14)). It is shown that, when microinhomogeneities are moving in a viscous medium, the structure of the frequency dependence of scattering (Eq. (20)) together with the parameters of spatial attenuation of low-frequency sound at $M > (ka)^2$ may considerably deviate from those described by the Rayleigh law. In particular, in a viscous microinhomogeneous medium, low-frequency sound attenuation is almost frequency independent. A viscous flow of the medium near inhomogeneities not only intensifies the sound absorption owing to additional loss, but also considerably enhances the scattered field. This refines the classical laws determining the effect of viscosity on the scattering's cross section in a microinhomogeneous medium, which are used for stationary inhomogeneities. On the basis of the data on the velocity and size distributions of falling drops of water in air, refinements are proposed for the laws of low-frequency sound attenuation due to the scattering of sound waves by rain.

REFERENCES

1. M. A. Isakovich, General Acoustics (Nauka, Moscow, 1973) [in Russian].
2. V. O. Knudsen et al., J. Acoust. Soc. Am. **20**, 849 (1948).
3. S. Kh. Rozenfel'd, Akust. Zh. **29**, 251 (1983) [Sov. Phys. Acoust. **29**, 145 (1983)].
4. F. A. Lyman, J. Sound Vibrat. **51**, 219 (1977).
5. V. I. Tatarskii, *Propagation of the Waves in Turbulent Atmosphere* (Nauka, Moscow, 1972) [in Russian].
6. A. M. Davis and R. J. Nagem, J. Acoust. Soc. Am. **119**, 2018 (2006).
7. L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics*, Vol. 6: *Hydrodynamics* (Nauka, Moscow, 1988; Pergamon, Oxford, 1975).
8. L. Prandtl and O. G. Tietjens, *Fundamentals of Hydro- and Aeromechanics* (McGraw-Hill, New York, 1934; Inostr. Liter., Moscow, 1949).
9. V. N. Alekseev, A. V. Rimskii-Korsakov, and A. G. Semenov, Akust. Zh. **38**, 581 (1992) [Sov. Phys. Acoust. **38**, 321 (1992)].
10. V. N. Alekseev and A. G. Semenov, Akust. Zh. **38**, 789 (1992) [Sov. Phys. Acoust. 3**8**, 433 (1992)].
11. V. N. Alekseev and A. G. Semenov, Akust. Zh. **39**, 197 (1993) [Acoust. Phys. **39**, 105 (1993)].
12. V. N. Alekseev and A. G. Semenov, Akust. Zh. **41**, 876 (1995) [Acoust. Phys. **41**, 774 (1995)].
13. V. N. Alekseev and A. G. Semenov, Akust. Zh. **46**, 732 (2000) [Acoust. Phys. **46**, 641 (2000)].
14. F. M. Morse and G. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill, New York, 1953; Inostr. Liter., Moscow, 1960).
15. D. I. Blokhintsev, *Acoustics in Moving Inhomogeneous Media* (Nauka, Moscow, 1981; Taylor Francis, London, 1998).
16. M. E. Goldstein, *Aeroacoustics* (McGraw-Hill, New York, 1976; Mashinostroenie, Moscow, 1981).

Translated by E. Golyamina