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CLASSICAL PROBLEMS OF LINEAR ACOUSTICS AND WAVE THEORY

Nonlinear and Linear Wave Phenomena in Narrow Pipes

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Abstract—Phenomena arising in the course of wave propagation in narrow pipes are considered. For nonlinear waves described by the generalized Webster equation, a simplified nonlinear equation is obtained that allows for low-frequency geometric dispersion causing an asymmetric distortion of the periodic wave profile, which qualitatively resembles the distortion of a nonlinear wave in a diffracted beam. Tunneling of a wave through a pipe constriction is investigated. Possible applications of the phenomenon are discussed, and its relation to the problems of quantum mechanics because of the similarity of the basic equations of the Klein–Gordon and Schrödinger types is pointed out. The importance of studying the tunneling of nonlinear waves and broadband signals is indicated.

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The Webster equation [1–3] describes the propagation of sound in pipes, horns, concentrators, and other waveguiding systems with variable cross sections $S(x)$. Here, x is the coordinate measured along the axis of the system. This equation is applicable to pipes the characteristic radius of which is small compared to the wavelength: $r_0(x) \ll \lambda$. In addition, the cross section should vary slowly: $dr_0/dx \ll 1$. This means that the tangent to the function describing the pipe profile $r_0(x)$ should make small angles with the x axis [3].

The generalized Webster equation appears in the problems of intense sound propagation in pipes [4–6]. It is also used for calculating the acoustic field in inhomogeneous media in the geometric acoustics approximation [4, 5], where it plays the role of the transfer equation represented in ray coordinates. The axis of a ray tube is the geometric ray calculated from the eikonal equation, and the function $S(x)$ is the cross section of the ray tube. We represent this equation in the form

$$\frac{\partial^2 p}{\partial t^2} - \frac{c^2}{S(x)}\frac{\partial}{\partial x}\left[S(x)\frac{\partial p}{\partial x}\right] = \frac{\varepsilon}{c^2\rho}\frac{\partial^2 p^2}{\partial t^2}. \qquad (1)$$

Here, p is the sound pressure, c is the velocity of sound, and ρ is the density of the medium. Let us introduce a new function F instead of the pressure:

$$p(x, t) = F(x, t)/\sqrt{S(x)}.$$

For this function, Eq. (1) takes the form

$$\frac{\partial^2 F}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 F}{\partial t^2} = \frac{1}{\sqrt{S(x)}}\frac{d^2\sqrt{S(x)}}{dx^2}F - \frac{1}{\sqrt{S(x)}}\frac{\varepsilon}{c^4\rho}\frac{\partial^2 F^2}{\partial t^2}. \qquad (2)$$

With the acoustic nonlinearity being ignored, Eq. (2) represents the Klein–Gordon equation with the coefficient depending on the x coordinate. If $S^{-1/2}(x)$ is given by one of the following functions:

$$C\sin\gamma(x+x_0), \quad C\cos\gamma(x+x_0), \quad C\sinh\gamma(x+x_0), \quad C\cosh\gamma(x+x_0), \quad C\exp[\pm\gamma(x+x_0)] \qquad (3)$$

(where C, x_0, and γ are constants), linearized equation (2) takes the form of the conventional Klein–Gordon equation

$$\frac{\partial^2 F}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 F}{\partial t^2} = \pm\gamma^2 F.$$

In this case, the wave acquires a low-frequency dispersion described by the law $k^2 = \omega^2/c^2 \pm \gamma^2$. In cases in which

$$\sqrt{S(x)} = C, \quad \sqrt{S(x)} = C(x+x_0),$$

i.e., for plane or spherically symmetric waves, the dispersion vanishes and the Klein–Gordon equation takes the form of the conventional wave equation.

Since, in the model described by Eqs. (1) and (2), the cross section varies slowly at distances on the order

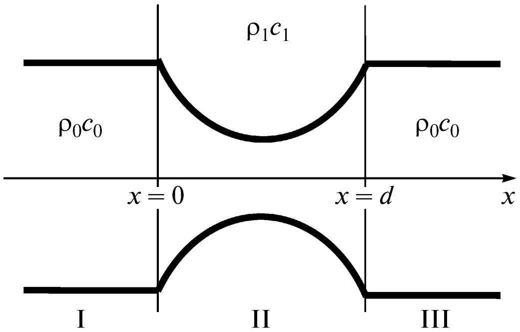

Fig. 1. A pipe with a constriction (region II, $0 < x < d$) filled with a medium that is characterized by the density ρ_1 and the sound velocity c_1.

of the wavelength and the nonlinearity is small, the above equation can be simplified for traveling waves. Using the slowly varying profile approach (see, e.g., [7]), for a wave traveling in the direction of increasing x coordinate values, Eq. (2) can be reduced to the form

$$\frac{\partial}{\partial\tau}\left[\frac{\partial F}{\partial x} - \frac{\varepsilon}{c^3\rho\sqrt{S(x)}}F\frac{\partial F}{\partial\tau}\right] = -\frac{c}{2\sqrt{S(x)}}\frac{d^2\sqrt{S(x)}}{dx^2}F. \quad (4)$$

Here, $\tau = t - x/c$ is the time in the coordinate system traveling with the wave with the velocity of sound. We note that the transition to the slowly varying profile approximation in the initial form of Eq. (1) leads to a less accurate equation for the sound pressure:

$$\frac{\partial p}{\partial x} + \frac{p}{2S}\frac{dS}{dx} - \frac{\varepsilon}{c^3\rho}p\frac{\partial p}{\partial\tau} = 0. \quad (5)$$

Equation (5) is equivalent to Eq. (4) with a zero right-hand side; i.e., it ignores the dispersion. However, Eq. (5) has a solution (see problem 7.11 in [8]) for any initial (at $x = 0$) shape of the wave profile $p(x = 0, t) = p_0\Phi(t)$:

$$p = p_0\sqrt{\frac{S(0)}{S(x)}}\Phi\left[\tau + \frac{\varepsilon}{c^3\rho}p\int_0^x\sqrt{\frac{S(0)}{S(x')}}dx'\right]. \quad (6)$$

At the same time, Eq. (4) can only be solved for some specific shapes of the pipe cross section $S(x)$ [9]. Solution (6) describes a wave with identical nonlinear distortions in the regions of positive and negative pressures, because $p(-\tau) = -p(\tau)$. In the presence of dispersion in Eq. (4), this symmetry fails. Therefore, if the initial signal is harmonic, a sawtooth-shaped wave is formed in the medium with the negative-pressure half-period being lengthened and smoothed and the positive-pressure half-period being shortened and possessing a higher amplitude [9]. Phase shifts arise between the harmonics, which were absent in solution (6). A similar behavior is observed for a wave in a diffracted beam; the diffraction leads to a similar low-frequency dispersion.

Thus, the weak dispersion due to the variation of the pipe cross section gives rise to qualitatively new effects in the behavior of a nonlinear wave.

Interesting phenomena arise in pipes even in the simplest case of linear approximation. One of the basic phenomena is the tunneling effect, which is still not clearly understood [10]. However, this effect is of great interest for acoustic and electromagnetic waves, as well as for waves of other natures. Owing to the analogy with the Schrödinger equation, the classical results should also be important for understanding the fine effects of particle transmission through a potential barrier in the problems of quantum mechanics.

We consider waves that are harmonic in time. For such waves, Eq. (2) without the nonlinear term has the form

$$\frac{d^2F}{dx^2} + \left(k_1^2 - \frac{1}{\sqrt{S(x)}}\frac{d^2\sqrt{S(x)}}{dx^2}\right)F = 0. \quad (7)$$

Let us consider one of the aforementioned (Eq. (3)) particular cases, namely, the case in which Eq. (7) takes the form of an equation with constant coefficients and has a simple general solution. Assuming that

$$S(x) = S_m\cosh^2\left[\gamma\left(x - \frac{d}{2}\right)\right], \quad \gamma = \frac{2}{d}\operatorname{arccosh}\frac{1}{\sqrt{S_m}}, \quad (8)$$

we reduce Eq. (7) to the form

$$\frac{d^2F}{dx^2} + [k_1^2 - \gamma^2]F = 0. \quad (9)$$

This case corresponds to the problem of wave propagation through the pipe constriction ($0 < x < d$) shown in Fig. 1. The region $0 < x < d$ is filled with a medium with the density ρ_1 and sound velocity c_1. Outside this region, the pipe is filled with another medium with the density ρ_0 and sound velocity c_0.

In Eqs. (7)–(9), $k_1 = \omega/c_1$; the constants S_m and d represent two geometric characteristics of the variable-thickness region: the minimal dimensionless area of the constriction, which is reached at $x = d/2$, and the length d of this region.

The wave tunneling regime corresponds to $\gamma^2 > k_1^2$. This regime is possible for low frequencies $k_1 < \gamma$ or

$$\omega < \frac{2c_1}{d}\operatorname{arccosh}\frac{1}{\sqrt{S_m}}, \quad k_1 d < 2\operatorname{arccosh}\frac{1}{\sqrt{S_m}} \equiv 2\varphi.$$

In this case, the complex amplitudes of pressure and particle velocity in region II corresponding to $0 < x < d$ are described by the expressions

$$p_{\mathrm{II}} = \frac{1}{\sqrt{S}}(P_+e^{-\mu x} + P_-e^{\mu x}),$$

$$u_{\mathrm{II}} = \frac{i}{k_1\rho_1 c_1\sqrt{S}} \times \left[P_+e^{-\mu x}\left(\mu + \frac{S'}{2S}\right) - P_-e^{\mu x}\left(\mu - \frac{S'}{2S}\right)\right]. \quad (10)$$

Here, we used the notation $\mu = \sqrt{\gamma^2 - k_1^2}$. In the other two regions, i.e., before the inhomogeneity ($x < 0$) and after it ($x > d$), the fields are given by the formulas

$$p_{\mathrm{I}} = P_i e^{ik_0x} + P_r e^{-ik_0x},$$
$$u_{\mathrm{I}} = \frac{1}{\rho_0 c_0}(P_i e^{ik_0x} - P_r e^{-ik_0x}), \quad (11)$$

$$p_{\mathrm{III}} = P_t e^{ik_0(x-d)}, \quad u_{\mathrm{III}} = \frac{1}{\rho_0 c_0} P_t e^{ik_0(x-d)}, \quad (12)$$

where the quantities P_i, P_r, and P_t are the amplitudes of the incident, reflected, and transmitted waves.

We assume that $S(0) = S(d) = 1$. Using Eq. (8) for the cross-sectional area, we determine the derivatives at the boundaries of the inhomogeneity:

$$\left.\frac{dS}{dx}\right|_{x=0} = -2b, \quad \left.\frac{dS}{dx}\right|_{x=d} = +2b,$$
$$b \equiv \frac{2}{d}\sqrt{1 - S_m}\,\mathrm{arccosh}\frac{1}{\sqrt{S_m}}.$$

To calculate the reflection and transmission coefficients for the wave, it is necessary to impose the continuity conditions on the pressure p and velocity u fields at the boundaries of the region $0 < x < d$:

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x}, \quad u = -\frac{i}{k\rho c}\frac{dp}{dx}.$$

Then, from Eqs. (10)–(12), we obtain the following expressions for the amplitudes:

$$P_i + P_r = P_+ + P_-, \quad (13)$$

$$P_i - P_r = i\frac{\alpha}{k_1}[P_+(\mu - b) - P_-(\mu + b)], \quad (14)$$

$$P_+e^{-\mu d} + P_-e^{\mu d} = P_t, \quad (15)$$

$$i\frac{\alpha}{k_1}[P_+e^{-\mu d}(\mu + b) - P_-e^{\mu d}(\mu - b)] = P_t. \quad (16)$$

We denote the ratio of the acoustic impedances of the two media as $\alpha = (\rho_0 c_0/\rho_1 c_1)$.

A convenient way of solving system of equations (13)–(16) is as follows. We first eliminate the reflected wave amplitude from Eqs. (13) and (14) and the transmitted wave amplitude from Eqs. (15) and (16). Then, we obtain the system of two equations

$$P_+\left[1 + i\frac{\alpha}{k_1}(\mu - b)\right] + P_-\left[1 - i\frac{\alpha}{k_1}(\mu + b)\right] = 2P_i, \quad (17)$$

$$P_+e^{-\mu d}\left[1 - i\frac{\alpha}{k_1}(\mu + b)\right] + P_-e^{\mu d}\left[1 + i\frac{\alpha}{k_1}(\mu - b)\right] = 0. \quad (18)$$

The solution to system of equations (17), (18) has the form

$$P_+ = \frac{2P_i}{\Delta}e^{\bar{\mu}}\left[1 + i\frac{\alpha}{\bar{k}_1}(\bar{\mu} - \bar{b})\right],$$

$$P_- = -\frac{2P_i}{\Delta}e^{-\bar{\mu}}\left[1 - i\frac{\alpha}{\bar{k}_1}(\bar{\mu} + \bar{b})\right], \quad (19)$$

$$\Delta = e^{\bar{\mu}}\left[1 + i\frac{\alpha}{\bar{k}_1}(\bar{\mu} - \bar{b})\right]^2 - e^{-\bar{\mu}}\left[1 - i\frac{\alpha}{\bar{k}_1}(\bar{\mu} + \bar{b})\right]^2.$$

Here, the overbar indicates the dimensionless quantities:

$$\bar{k}_1 = k_1 d, \quad \bar{\mu} = \mu d = \sqrt{4\varphi^2 - \bar{k}_1^2},$$
$$\bar{b} = bd = 2\varphi\tanh\varphi. \quad (20)$$

Below, for brevity, we omit the overbar in dimensionless quantities (20). Using Eqs. (18) and (19), we can easily calculate the reflection and transmission coefficients for the wave:

$$R = \frac{P_r}{P_i} = \frac{e^{\mu}\left[1 + \frac{\alpha^2}{k_1^2}(\mu - b)^2\right] - e^{-\mu}\left[1 + \frac{\alpha^2}{k_1^2}(\mu + b)^2\right]}{e^{\mu}\left[1 + i\frac{\alpha}{k_1}(\mu - b)\right]^2 - e^{-\mu}\left[1 - i\frac{\alpha}{k_1}(\mu + b)\right]^2}, \quad (21)$$

$$T = \frac{P_t}{P_i} = \frac{i\frac{4\alpha\mu}{k_1}}{e^{\mu}\left[1 + i\frac{\alpha}{k_1}(\mu - b)\right]^2 - e^{-\mu}\left[1 - i\frac{\alpha}{k_1}(\mu + b)\right]^2}. \quad (22)$$

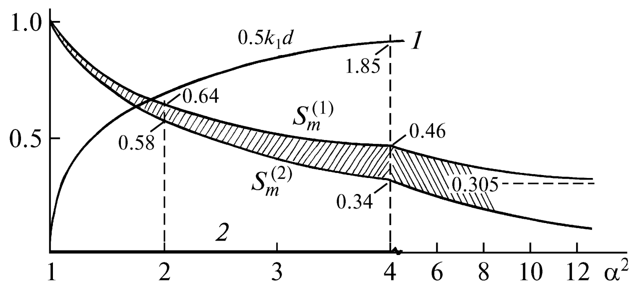

Fig. 2. The upper curve $S_m^{(1)}$ shows the dependence of the minimal cross section in the inhomogeneity constriction on the square of the impedance ratio of two media α^2; it corresponds to curve *1* for k_1d. The lower curve $S_m^{(2)}$ corresponds to $k_1d = 0$ (straight line *2*). The range of minimal cross section variation within which a total transmission is possible for two frequency values and an almost total transmission in the frequency band between the two maxima of the transmission coefficient is indicated by the hatched area.

Expressions (21) and (22) satisfy the law of conservation of energy:

$$|R|^2 + |T|^2 = 1. \tag{23}$$

In addition, since the numerator of Eq. (21) is purely real while the numerator of Eq. (22) is purely imaginary, the phases of the reflected and transmitted waves are related to each other:

$$\Phi_r - \Phi_t = -\frac{\pi}{2}.$$

Indeed, from calculations, we obtain the expressions

$$|R|^2|\Delta|^2 = \left\{e^{\mu}\left[1 + \frac{\alpha^2}{k_1^2}(\mu - b)^2\right]\right.$$

$$\left.- e^{-\mu}\left[1 + \frac{\alpha^2}{k_1^2}(\mu + b)^2\right]\right\}^2, \quad |T|^2|\Delta|^2 = 16\frac{\alpha^2}{k_1^2}\mu^2,$$

$$|\Delta|^2 = 16\frac{\alpha^2}{k_1^2}\mu^2 \tag{24}$$

$$+ \left\{e^{\mu}\left[1 + \frac{\alpha^2}{k_1^2}(\mu - b)^2\right] - e^{-\mu}\left[1 + \frac{\alpha^2}{k_1^2}(\mu + b)^2\right]\right\}^2,$$

which allow us to verify the fulfillment of conservation law (23). The phases of the reflection and transmission coefficients are determined by the formulas

$$\Delta = |\Delta|\exp(i\Phi),$$

$$\Phi = \arctan\left[2\frac{\alpha}{k_1}\right.$$

$$\left.\times \frac{e^{\mu}(\mu - b) + e^{-\mu}(\mu + b)}{e^{\mu}\left[1 - \frac{\alpha^2}{k_1^2}(\mu - b)^2\right] - e^{-\mu}\left[1 - \frac{\alpha^2}{k_1^2}(\mu + b)^2\right]}\right],$$

$$R = |R|\exp(-i\Phi), \quad T = |T|\exp(-i\Phi + \pi/2).$$

It is of interest to consider the case in which the reflected wave is absent (a total transmission), which, according to Eq. (21), is possible under the condition

$$\exp(2\mu) = \frac{k_1^2 + \alpha^2(\mu + b)^2}{k_1^2 + \alpha^2(\mu - b)^2}. \tag{25}$$

Using Eqs. (20), we represent condition (25) in the form

$$\exp(2\sqrt{4\varphi^2 - k_1^2}) = \frac{k_1^2 + \alpha^2(\sqrt{4\varphi^2 - k_1^2} + 2\varphi\tanh\varphi)^2}{k_1^2 + \alpha^2(\sqrt{4\varphi^2 - k_1^2} - 2\varphi\tanh\varphi)^2}. \tag{26}$$

Here, for brevity, we used the notation $\varphi(S_m) = \operatorname{arccosh}(S_m^{-1/2})$.

It should be noted that, at $\alpha = 1$, i.e., when the acoustic impedances of the two media are identical, condition (26) takes the form

$$\frac{\tanh(\sqrt{4\varphi^2 - k_1^2})}{\sqrt{4\varphi^2 - k_1^2}} = \frac{\tanh(2\varphi)}{2\varphi}. \tag{27}$$

Equation (27) has a single solution $k_1 = 0$, which corresponds to the pipe constriction characterized by zero wave thickness (or a wave with zero frequency). A nontrivial root only appears when the impedances of the media are different ($\alpha \neq 1$).

At fixed parameters of the media (a fixed value of $\alpha > 1$), Eq. (26) determines the implicit dependence of k_1^2 on φ. The root of this equation determines the dependence of the frequency corresponding to the total transmission of the wave on the geometric parameters of the pipe constriction region, S_m and d.

The results of analyzing Eq. (26) are presented in Fig. 2. The upper curve $S_m^{(1)}$ represents the dependence of the minimal cross section of the constriction on the square of the impedance ratio of the two media,

α^2. This dependence monotonically decreases from unity (at $\alpha^2 = 1$) to a value of 0.305, which is determined by the condition

$$S_m^{(1)} = \cosh^{-2}\varphi_*, \quad \varphi_* \approx 1.2, \quad \varphi_* \tanh\varphi_* = 1.$$

Choosing a certain value of α^2 and the corresponding minimal cross section $S_m^{(1)}$, we use curve *1* to determine the dimensionless frequency $\bar{k}_1 = k_1 d$. The second curve $S_m^{(2)}$ describes a monotonically decreasing (from unity to zero) function of α^2; these values of the minimal cross section correspond to the zero frequency value: $\bar{k}_1 = k_1 d = 0$ (straight line *2*).

The hatched area in Fig. 2 represents the range of minimal cross section variation within which a total transmission of the wave is possible for two frequency values and an almost total transmission for the frequencies lying in the frequency band between the two maxima of the transmission coefficient.

Thus, for a given value of α^2 (the parameters of the media are preset), from Fig. 2 we determine the range of values of S_m. Then, we choose a specific value of S_m; in this way, the geometry of the inhomogeneity is fixed. Finally, we calculate the value of $\bar{k}_1 = k_1 d$ (which uniquely determines the frequency) lying between curve *1* and straight line *2*.

To illustrate the use of the plot shown in Fig. 2, we take a fixed value of α^2; for example, let $\alpha^2 = 4$. Moving along the vertical dashed line from the abscissa axis to the intersection with the curve $S_m^{(2)}$, we observe an increase in the minimal cross section S_m from zero to 0.34. In this range of values of S_m, the transmission coefficient has a single maximum, which corresponds to unity and is reached at the dimensionless frequency $k_1 d = 0$. When the cross section reaches the value $S_m = 0.34$, a second maximum $|T| = 1$ appears in the region $k_1 d > 0$. Between the two maxima, a “plateau” is formed, where $|T| \approx 1$ in a finite frequency band. This situation persists until the intersection with the curve $S_m^{(1)}$, where the cross-sectional area becomes $S_m = 0.46$. At this point, the second maximum $|T| = 1$ falls on the boundary of the region within which solution (24) is valid. This qualitative analysis on the basis of Fig. 2 is confirmed by the results of numerical calculation of the transmission coefficient (Fig. 3).

The calculation was performed on the basis of the general expressions for the coefficient of wave transmission through the combined inhomogeneity shown

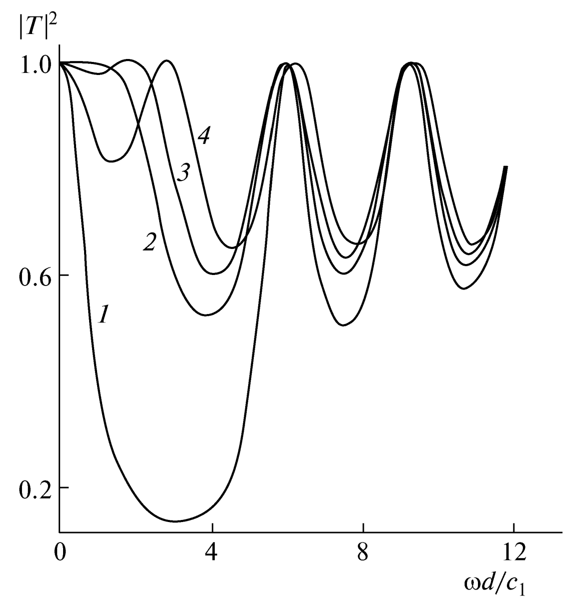

Fig. 3. Frequency dependence of the square of the wave transmission coefficient for $\alpha = 2$ and four different values of the minimal pipe cross section: $S_m = 0.09$, 0.34, 0.46, and 0.6 (curves *1*–*4*, respectively).

in Fig. 1. The expressions can be represented in the form

$$|T(k_1)|^{-2} = 1 + \frac{k_1^2}{16\alpha^2\mu^2}$$

$$\times \left\{ e^{\mu}\left[1 + \frac{\alpha^2}{k_1^2}(\mu - b)^2\right] - e^{-\mu}\left[1 + \frac{\alpha^2}{k_1^2}(\mu + b)^2\right]\right\}^2, \quad (28)$$

$$\mu = \sqrt{4\varphi^2 - k_1^2}, \quad 0 < k_1 < 2\varphi,$$

$$|T(k_1)|^{-2} = 1 + \frac{k_1^2}{4\alpha^2\nu^2}$$

$$\times \left\{\left[1 + \frac{\alpha^2}{k_1^2}(b^2 - \nu^2)\right]\sin\nu - 2\frac{\alpha^2}{k_1^2}\nu b\cos\nu\right\}^2, \quad (29)$$

$$\nu = \sqrt{k_1^2 - 4\varphi^2}, \quad 2\varphi < k_1 < \infty.$$

Expressions (28) and (29) are continuously sewed together at the point $k_1 = 2\varphi$. At higher dimensionless frequencies $k_1 \gg 2\varphi$, Eq. (29) transforms to the well-known solution (see, e.g., [11]) describing oscillations of the transmission coefficient $|T(k_1)|$. The frequency dependence of the transmission coefficient determined by Eqs. (28) and (29) is represented in Fig. 3. Here, the ratio of acoustic impedances was taken to be $\alpha = 2$ and, for the minimal cross section of the pipe constriction, we took the values $S_m = 0.09$, 0.34, 0.46, and 0.6.

It would be of interest to solve the problem of the broadband pulse propagation through an inhomogeneity possessing a region of weak variation of the coefficient $|T(k_1)|$. Such a region may occur, in particular, for the parameters $S_m = 0.46$ and $\alpha = 2$ (curve *3* in Fig. 3). If, at the input, the spectrum of the pulsed signal is bounded and lies in the low-frequency range, for example, $0 < \omega < 2c_1/d$, where the transmission coefficient is close to unity (for curve *3* in Fig. 3), the pulse can tunnel through the inhomogeneity by undergoing phase distortions only.

At large values of $\alpha^2 \gg 1$ (this may be a gas layer surrounded by a condensed medium) and small cross sections S_m, for the frequency corresponding to total transmission, from Eq. (26) we obtain a simple approximate formula:

$$f = \frac{\sqrt{2}}{2\pi}\frac{c_1}{d}\sqrt{S_m}\ln\frac{4}{S_m}.$$

For example, for $d = 0.33$ cm, $S_m = 0.1$, and an air layer with $c_1 = 330$ m/s, from Eq. (25) we obtain the frequency $f = 26$ kHz.

The tunneling phenomenon described above may presumably be used in the design of frequency-selective filters with controlled characteristics. By analogy with laser physics, one can consider the development of a "gate" or a modulator of Q-factor for an acoustic resonator. If, at the initial instant of time, the constriction of the pipe is completely closed, i.e., $S_m = 0$, the wave cannot be transmitted through the inhomogeneity and is completely reflected inside the resonator, where considerable energy can be accumulated if the Q-factor is sufficiently high. Then, if S_m increases sufficiently rapidly (e.g., with the help of a piezoelectric element) up to the value providing the tunneling regime, the Q-factor will drastically decrease and the acoustic energy will be released. This process is similar to the operation of a Kerr shutter in a Q-switched laser.

In closing, we note that this paper is intended to attract the attention of researchers to the problem of wave tunneling through special profiles of pipe constrictions. In the context of this problem, it is of interest to study the propagation of broadband signals, namely, pulses and nonlinear waves. Our paper may serve as a "sketch" for subsequent investigations.

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