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NONLINEAR ACOUSTICS

Characteristic Features of Elastic Wave Propagation in a One-Dimensional Model of an Unconsolidated Medium[1]

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Abstract—The characteristic features of elastic wave propagation in a one-dimensional model of a discrete inhomogeneous unconsolidated medium are investigated. The model is represented by a linear chain of 80 uncoupled steel spheres with a diameter of 6.5 mm. Nonlinear effects that may arise in such systems are reviewed. The experimental setup is described. Results of studying the dispersion of elastic waves in the system and the dependence of the elastic wave velocity on the wave amplitude under increasing compression are presented. The results are analyzed using the Hertz contact theory.

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INTRODUCTION

In acoustics, one of the topical areas of research close to geophysics and materials science is represented by studies of nonlinear processes caused by the presence of mesoscale inhomogeneities and defect structure in solids. Inhomogeneities considerably affect the elastic properties of media and give rise to new physical properties that are absent in homogeneous solids [1, 2]. Among structurally inhomogeneous materials, a special place is occupied by granular media because of their interesting physical properties and their widespread occurrence in nature. A comprehensive review of the results of studying the excitation and propagation of seismic waves in discrete media can be found in [3].

The theory of elastic properties of unconsolidated granular media is based on the problem of contact interaction between individual grains. The area of contacts between grains depends on the applied stress; i.e., the system is deformed as an ensemble of nonlinear springs. For spherical bodies experiencing elastic deformation, the problem was formulated and solved by Hertz (see, e.g., [4, 5]).

In the particular case of interaction between two spheres of radius R, the radius of the contact spot a is expressed through the static compression force F and the reduced elastic modulus E^* as

$$a = \left(\frac{3FR}{4E^*}\right)^{1/3}, \quad \frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}, \qquad (1)$$

where $E_{1,2}$ are Young's moduli of the spheres and $\nu_{1,2}$ are their Poisson's ratios. The mutual approach of the spheres is

$$h_0 = \left(\frac{9F^2}{16RE^{*2}}\right)^{1/3}, \qquad (2)$$

and the force F driving them toward each other is

$$F = \frac{4E^*\sqrt{R}}{3}h_0^{3/2}. \qquad (3)$$

When the difference h_0 between the displacements of the sphere centers is negative, the spheres move away from each other without deformation and the force F is zero. When the aforementioned difference is positive, the force F varies as $h_0^{3/2}$. It is evident that, if a system consisting of two spheres oscillates under a periodic external force, elastic nonlinearity will be noticeable only in the case of considerable variations in the contact area of the bodies. Therefore, as the compression force increases, the nonlinearity in the contact region decreases. In the case of a weak compression, when stretching forces can break the contact, the grains collide. This mechanism is called "clapping" nonlinearity [6].

Modeling of a structurally inhomogeneous medium by a chain of spheres was described in [7]. The author studied nonstationary nonlinear perturbations. The characteristic times τ of the problem were much greater than the period T of the sphere shape oscillations:

$$\tau \gg T \approx 2.5R/C_1,$$

where C_1 is the velocity of sound in the sphere material. A numerical study was performed for a one-dimensional chain of identical spherical grains. The chain was loaded with a static compression force F_0, which was applied to the ends of the chain and caused the initial mutual approach of the sphere centers h_0. For an individual grain of number i, a displacement u_i from the equilibrium position was introduced. For such a particle, the equation of motion has the form

$$\ddot{u}_i = A(h_0 - u_i + u_{i-1})^{3/2} - A(h_0 - u_{i+1} + u_i)^{3/2}, \quad (4)$$

$$A = \frac{E\sqrt{2R}}{3m(1-\nu^2)}, \quad N-1 \geq i \geq 2, \quad (5)$$

where m is the mass of a single grain. Here, it is assumed that the distance between the centers of particles does not exceed $2R$. Equation (4) can also describe the propagation of one-dimensional perturbations in a three-dimensional simple cubic packing of spheres if the front plane is parallel to the cube faces. A similar form of equations of motion is suitable for other regular packings. The differences only manifest themselves in the numerical coefficient A involved in Eq. (5).

Equation (4) can be reduced to the well-known equations describing a system of nonlinear oscillators under the assumption that the deformations in the medium are small compared to the initial mutual approach h_0:

$$|u_{i-1} - u_i|/h_0 \ll 1.$$

In the lowest anharmonic approximation, Eq. (4) has the form

$$\begin{aligned} \ddot{u}_i &= \alpha(u_{i+1} - 2u_i + u_{i-1}) \\ &+ \beta(u_{i+1} - 2u_i + u_{i-1})(u_{i-1} - u_{i+1}), \\ \alpha &= \frac{3}{2}Ah_0^{1/2}, \quad \beta = \frac{3}{8}Ah_0^{-1/2}, \quad N-1 \geq i \geq 2. \end{aligned} \quad (6)$$

In the long-wave limit, Eq. (6) yields the nonlinear wave equation [7]

$$\begin{aligned} U_{tt} - C_0^2 U_{xx} &= 2C_0\gamma U_{xxxx} - \varepsilon U_x U_{xx}, \\ C_0^2 = 6AR^2\sqrt{h_0}, \quad \gamma &= C_0 R^2/6, \quad \varepsilon = C_0^2 R/h_0. \end{aligned} \quad (7)$$

In [7], the author analyzed the soliton solution to Eq. (7), which satisfied the Korteweg–de Vries equation accurate to the terms quadratic in the nonlinearity ε and dispersion γ coefficients. The velocity proved to be proportional to the fourth root of the initial approach, $c_0 \sim h_0^{1/4}$, or, in terms of the force, $c_0 \sim F^{1/6}$, and the nonlinearity behaved as $\varepsilon \sim 1/h_0$.

Beginning with the publications by V.F. Nesterenko, numerous experimental and theoretical studies of one-dimensional granular media were carried out, many of them being devoted to studying solitons. For example, in [8], the propagation of large-amplitude compression pulses in a one-dimensional chain of steel spheres under weak static compression was investigated. All of the experimental observations of single pulses (solitons) in such a chain were in good agreement with theoretical results [7]. In [9], self-modulation processes in a granular chain were considered with allowance for the dispersion due to the discrete nature of the medium. In [10], an ensemble of grains immersed in a liquid was studied. The system of grains itself possesses a high structural nonlinearity due to the boundary contacts. In the presence of an oscillating liquid, an additional inertial nonlinearity appears because of the accelerated motion of particles. Attraction forces arise between the particles in a liquid flow, and the Hertz repulsion manifests itself in the course of the deformation of colliding grains. In this case, large spatial gradients of forces are caused by the inhomogeneity of mass distribution. In such a medium, the natural frequency of linear oscillations of an elementary oscillator is

$$f_{\text{lin}} = \frac{1}{2\pi}\sqrt{\frac{3a}{2}}\left(E^2 F_0 \frac{R_1 R_2}{R_1 + R_2}\right)^{1/6},$$

where a is the coefficient depending on the volumes and densities of two neighboring spherical particles and F_0 is the static compression force. For large oscillation amplitudes, when the amplitude A is on the order of the grain diameter, the nonlinear frequency [10]

$$f_{\text{nonlin}} = \frac{\sqrt{a}F_0^{5/6}}{2AE^{1/3}}\left(\frac{R_1 + R_2}{R_1 R_2}\right)$$

proves to be two to three orders of magnitude smaller than the frequency in the linear case.

If an acoustic wave propagates in the liquid surrounding the grains, the spheres are attracted to each other, collide, and move apart, their motion in the liquid being almost free. After subsequent collisions, the spheres move progressively farther from one another. Since, at the instant of collision, the relative velocities of neighboring spheres are random, a random oscillation pattern is formed. The oscillation amplitude on the average increases with time, and the spectrum of oscillations shifts to lower frequencies [10].

Three-dimensional structures representing natural granular media, such as sand or pebble, are of practical interest for studies and diagnostics. In [11], slow nonlinear acoustic processes were studied in hysteretic granular media. For this purpose, a granite aggregate was used with a grain size of 1–2 cm. Sound was generated at a frequency of 5.6 kHz by a piezoceramic plate and received by accelerometers, which were placed among the grains and had the size close to that of grains. The dependence of the oscillation amplitude of an individual grain on the amplitude of acoustic signal excitation was found to be nonmonotonic. The

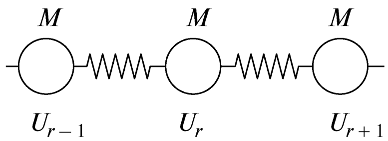

Fig. 1. Chain of identical masses connected with springs.

spectra of slow fluctuations of harmonic and subharmonic components of the signal propagating in the granular medium were investigated.

In [12], nonlinear effects arising in a three-dimensional granular medium were studied. Demodulation of amplitude modulated longitudinal and shear waves was observed. As a result of demodulation, a longitudinal wave was formed irrespective of the pumping wave polarization.

In addition to the giant nonlinearity providing the high sensitivity of nonlinear measuring techniques, structurally inhomogeneous media are also of interest because of the unusual nonlinear phenomena observed in them [1]. One of these phenomena is the presence of the "dominant" frequency in the media of the type of moist sand, clay, or cracked rock. No matter what the oscillation excitation frequency may be, at the output of such a medium, the "dominant" signal is received while other spectral components, including the initial frequency, are weak. The characteristic values of the dominant frequency are as follows: 8–10 Hz for gravel, 25 Hz for sea sand, 40 Hz for clay, and 100 Hz for eroded granite. It is of interest that, when vibrations with a dominant frequency of 12 Hz are applied to a water-encroached oil pool, the oil fraction in the discharge approximately doubles. The origin of the dominant frequencies is the presence of internal resonance properties of fragmented soils and rocks, as well as their strong nonlinearity causing the oscillation energy transfer to these frequencies.

Today, granular media and, in particular, one-dimensional chains of balls, are studied by several research groups. Most of the studies are devoted to solitary waves (solitons) in these media. Periodic waves are less investigated. Therefore, the purpose of this paper is the study of the characteristic features of periodic wave propagation in a one-dimensional model of a granular medium.

THE BASIC FORMULAS AND ESTIMATES

Let us consider a chain of spheres that have identical masses and are connected with springs [13, 14]. For each element of the chain (Fig. 1), we introduce the displacement from the equilibrium position, $U_r \ll h_0$, where h_0 is the initial mutual approach of the sphere centers, r is the number of a particle, a is the distance between the elements, M is the mass of a particle, and μ is the coefficient of elasticity of a spring. The system of equations of motion has the form

$$M\frac{\partial^2 U_r}{\partial t^2} = F_{r-1} - F_{r+1} \tag{8}$$
$$= \mu(U_{r-1} - U_r) - \mu(U_r - U_{r+1}) = \mu(U_{r-1} + U_{r+1} - 2U_r).$$

For propagating waves, the solution is sought in the form

$$U_r = e^{i(\omega t - kar)}, \tag{9}$$

where ω is the cyclic frequency, $k = \omega/V_{ph} = 2\pi/\lambda$ is the wave number, V_{ph} is the phase velocity of the wave in the chain, and λ is the wavelength. Substituting Eq. (9) in Eq. (8), we obtain the dispersion relation

$$\omega = 2\sqrt{\frac{\mu}{M}}\sin\left(\frac{ka}{2}\right). \tag{10}$$

To analyze small-amplitude waves in the system with Hertzian nonlinearity, we use the above dispersion relation, in which the elastic modulus μ depends on the initial compression. In the case of a Hertz contact, the force F is related to compression by nonlinear relation (3):

$$F = \mu h_0^{3/2},$$

which yields

$$\mu = \frac{\partial F}{\partial h_0} = \frac{E\sqrt{R}}{(1-\nu^2)}h_0^{1/2}. \tag{11}$$

With allowance for Eq. (11), dispersion relation (10) for the chain of spheres with Hertz contacts takes the form

$$\omega = ka\sqrt{\frac{E\sqrt{R}}{(1-\nu^2)M}h_0^{1/2}}\frac{\sin(ka/2)}{(ka/2)}.$$

This allows us to obtain the expressions for the phase V_{ph} and group V_{gr} velocities:

$$V_{ph} = \frac{\omega}{k} = a\sqrt{\frac{E\sqrt{R}}{(1-\nu^2)M}h_0^{1/2}}\frac{\sin(ka/2)}{(ka/2)}, \tag{12}$$

$$V_{gr} = \frac{\partial\omega}{\partial k} = a\sqrt{\frac{E\sqrt{R}}{(1-\nu^2)M}h_0^{1/2}}\cos(ka/2). \tag{13}$$

In Eqs. (12) and (13), we substitute the distance between the particles $a = 2R$ and the expression for h_0 (Eq. (2)) from the Hertz contact theory,

$$h_0 = \left(\frac{3F(1-\nu^2)}{2E\sqrt{R}}\right)^{2/3}.$$

As a result, we obtain the dependence of the phase V_{ph} and group V_{gr} velocities in the chain and the dispersion dependence $\omega(k)$ on the force F:

$$V_{ph} = \frac{\omega}{k} \frac{2R}{\sqrt{M}} \left(\frac{3F}{2}\right)^{1/6} \left(\frac{E\sqrt{R}}{(1-\nu^2)}\right)^{1/3} \left(\frac{\sin(kR)}{kR}\right), \quad (14)$$

$$V_{gr} = \frac{\partial\omega}{\partial k} = \frac{2R}{\sqrt{M}} \left(\frac{3F}{2}\right)^{1/6} \left(\frac{E\sqrt{R}}{(1-\nu^2)}\right)^{1/3} \cos(kR), \quad (15)$$

$$\omega = \frac{2}{\sqrt{M}} \left(\frac{3F}{2}\right)^{1/6} \left(\frac{E\sqrt{R}}{(1-\nu^2)}\right)^{1/3} \sin(kR). \quad (16)$$

From Eqs. (14)–(16), it follows that both phase V_{ph} and group V_{gr} velocities exhibit a dispersion. At $kR = 2\pi R/\lambda = \pi/2$ or $\lambda = 4R$, the group velocity V_{gr} is zero. The system under consideration has a maximal frequency ω_{max}, above which the elastic wave cannot propagate in the system; i.e., the system operates as a low-pass filter:

$$\omega_{max} = \frac{2}{\sqrt{M}} \left(\frac{3F}{2}\right)^{1/6} \left(\frac{E\sqrt{R}}{(1-\nu^2)}\right)^{1/3}. \quad (17)$$

From the analysis of Eqs. (14)–(17), it follows that the physical parameters of the chain of spheres, namely, the phase V_{ph} and group V_{gr} velocities and the cutoff frequency in the periodic discrete system with Hertzian nonlinearity, can be controlled by an external static force.

Substituting the known quantities, i.e., the parameters of steel [19] and the parameters of the chain (see table), in the above formulas, we calculate the dependence of the maximal frequency of the wave (the cutoff frequency) as a function of the force applied to the chain:

$$\omega_{max} = \frac{2}{\sqrt{M}} \left(\frac{3F}{2}\right)^{1/6} \left(\frac{E\sqrt{R}}{(1-\nu^2)}\right)^{1/3} \approx 145400 F^{1/6} \text{ rad/s},$$

$$\text{or } f_{max} = 23 F^{1/6} \text{ kHz}.$$

The parameters of the chain of balls used in the experiment

Number of chain elements (balls)	$N = 80$
Material	Steel
Ball radius	$R = 3.25$ mm
Ball mass	$m = 1.12$ g
Surface roughness (rms deviation of the surface profile)	$R_{mean\ dev.} \approx 1$ μm
Maximal allowable compression force for the system to remain in the elastic deformation region	$F_{max} = 120$ N
Maximal contact radius	$A_{max} = 138.89$ μm
Maximal contact area	$S_{max} = 0.06$ mm^2
Maximal mutual approach of two ball centers	$h_{0\ max} = 5.93$ μm
Length of the chain consisting of 80 balls	$\ell = 52$ cm
Maximal decrease in the chain length	$\Delta\ell_{max} = 0.47$ mm

DESCRIPTION OF THE EXPERIMENT

To study the characteristic features of elastic wave propagation in a periodic unconsolidated structure with a strong Hertzian contact nonlinearity [15–17], we used the experimental setup schematically represented in Fig. 2. The medium was modeled by a chain of 80 identical steel balls 6.5 mm in diameter; the balls were placed in a fabric-based laminate tube. The gap between the balls and the tube was small to maintain the alignment of the system; on the other hand, it was sufficiently large to allow free motion of the balls. Unfortunately, we could not completely eliminate dry friction. The table shows the main parameters of the system used in the experiment and the limitations imposed on the external conditions because of the fact that the Hertz contact theory is only valid in the elastic deformation region. The main parameters of the contact region that were calculated on the basis of the Hertz theory [4] are also presented in the table.

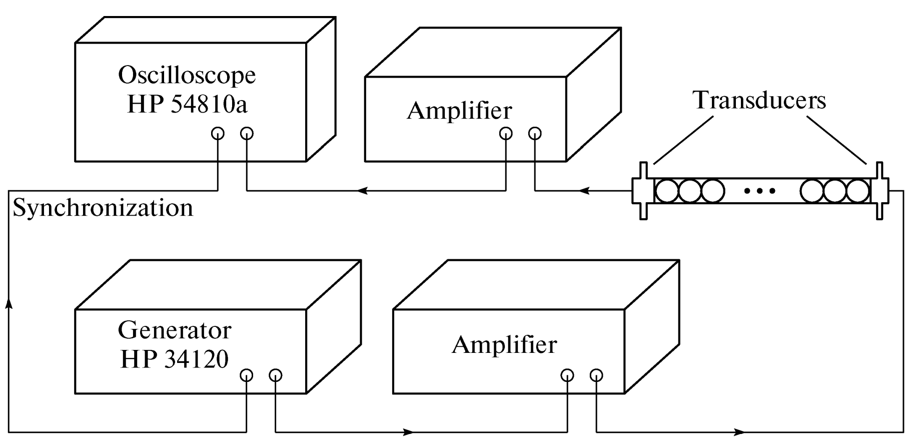

Fig. 2. Flow chart of the experimental setup.

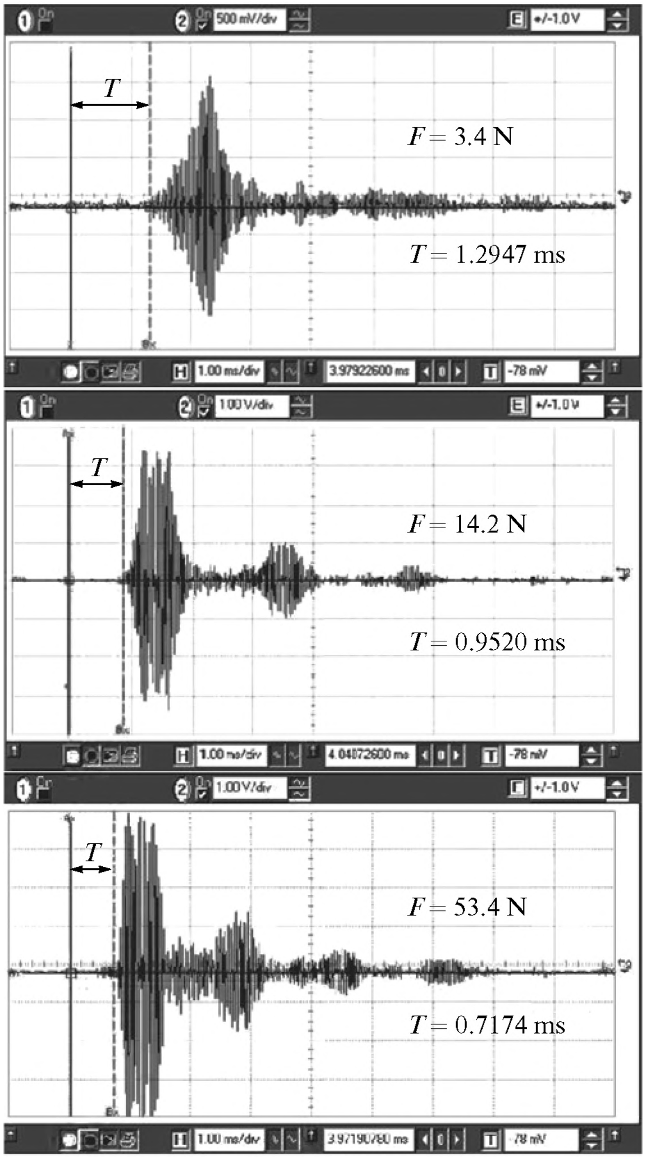

Fig. 3. Oscillograms of the acoustic signal in the chain of balls for different values of the compression force. (The solid mark indicates the onset of the sounding pulse.)

For longitudinal wave excitation in the chain, we used a compound transducer, which consisted of four piezoceramic rings with a thickness of 1.7 mm and a diameter of 10.9 mm. A static compression force was applied to the outer side of the radiating transducer. At the other end of the chain, elastic waves were received by a GS0506HR transducer (Matec Instruments, Inc.) with a resonance frequency of 5 MHz. The receiving transducer had a uniform amplitude–frequency characteristic in the frequency band 0.01–3 MHz. The acoustic contact between the transducer and the system occurred through salol. Radio pulses with a duration of 10–16 periods of the carrier frequency were sent from an HP 34120 generator to a power amplifier (Behringer Europower 2500). The amplified sounding radio pulses, the amplitude of which could be varied within 0 to 100 V, were supplied to the radiating transducer. The signal transmitted through the chain of balls was amplified by a weak-signal amplifier, observed with a dual-beam oscilloscope (HP 54810a), and sent to a computer for storage and further data processing.

The sound velocity in the one-dimensional chain of balls was measured by the echo pulse method [18]. Measuring the elastic pulse propagation time, which was identical to the delay τ_d between the sounding pulse and the pulse transmitted through the chain with a length l, it was possible to determine the velocity of sound,

$$C = l/\tau_d.$$

The errors in velocity measurements were mainly determined by the errors in measuring the time τ_d. Estimates showed that the errors related to variations in the chain length under the static force can be ignored.

Figure 3 shows a series of acoustic pulses at a frequency of 24 kHz that were observed by the oscilloscope for different values of the external compression force F. One can see that an increase in F leads to a considerable decrease in the propagation time of the elastic wave, i.e., to an increase in its velocity. For example, as the force F varies from 3.4 to 53.4 N, the velocity increases by a factor of 1.8. As one can see from Fig. 3, an increase in the compression force is accompanied by a decrease in the elastic wave attenuation in the chain. According to [17], the attenuation of elastic waves in the given system is determined not by the attenuation in the material of the balls (steel), but by the area of their contacts. From Eq. (1), one can see that, as the compression force increases, the radius of the ball contact area increases. This reduces the sound attenuation in the chain. Despite the considerable decrease in the elastic wave attenuation in the chain of balls with an increase in compression, the attenuation remains much greater than the wave attenuation in steel at a frequency of 24 kHz [19].

We also measured the frequency dependence of the group velocity, $V_{gr} = V_{gr}(\omega)$, in the chain for different values of the compression force. Figure 4 shows the experimental results superimposed on the theoretical dependences, which were calculated using Eqs. (16) and (17).

The dispersion curves were calculated as follows. According to the definition of the group velocity, we have

$$\omega = \int V_{gr} dk. \tag{18}$$

Substituting Eq. (15) for V_{gr} in Eq. (18) and performing some transformations, we obtain the expression for $\omega(k)$,

$$\omega(k) = V_{gt}\tan(kR)/R.$$

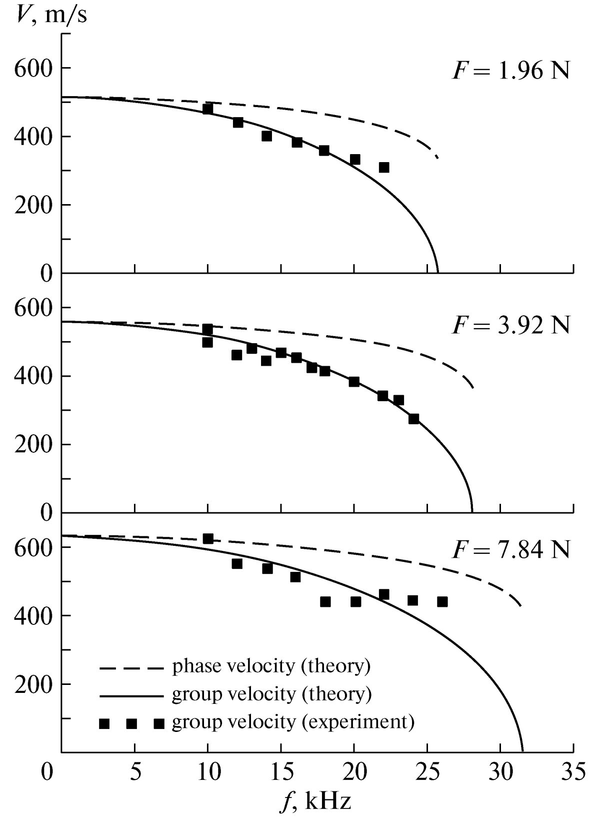

Fig. 4. Dependences of the phase and group velocities on the frequency of the elastic wave.

For the same compression forces, theoretical dispersion curves $\omega(k)$ were plotted (the solid line) and experimental points were superimposed on them (Fig. 5). As one can see from Figs. 4 and 5, the experimental results agree fairly well with the theoretical calculations. The deviations observed in the plot are related to the presence of friction between the balls and the tube and also to the neglect of the finite size of the balls and their surface roughness in the calculations.

Now, let us analyze the dependence of the elastic wave velocity on the wave amplitude for different values of the static compression force. We consider a harmonic wave propagating in the chain and assume that the wave amplitude h satisfies the condition $h \ll h_0$. Then, Eq. (15) can be represented in the form

$$V_{gr} = \frac{\partial\omega}{\partial k} = a\sqrt{\frac{E\sqrt{R}}{(1-\nu^2)M}}(h_0+h)^{1/2}\cos(ka/2) = C(h+h_0)^{1/4} = Ch_0^{1/4}(1+\varepsilon)^{1/4}, \quad (19)$$

where $C = a\sqrt{\frac{E\sqrt{R}}{(1-\nu^2)M}}\cos(ka/2)$ and $\varepsilon = (h/h_0) \ll 1$. Now, In Eq. (19), we expand $(1 + \varepsilon)^{1/4}$ in a Taylor series and restrict it to terms quadratic in ε. This yields

$$V_{gr} = \frac{\partial\omega}{\partial k} = C(h+h_0)^{1/4} = Ch_0^{1/4}(1+\varepsilon)^{1/4} = Ch_0^{1/4}\left[1+\frac{1}{4}\varepsilon-\frac{3}{32}\varepsilon^2+\ldots\right]. \quad (20)$$

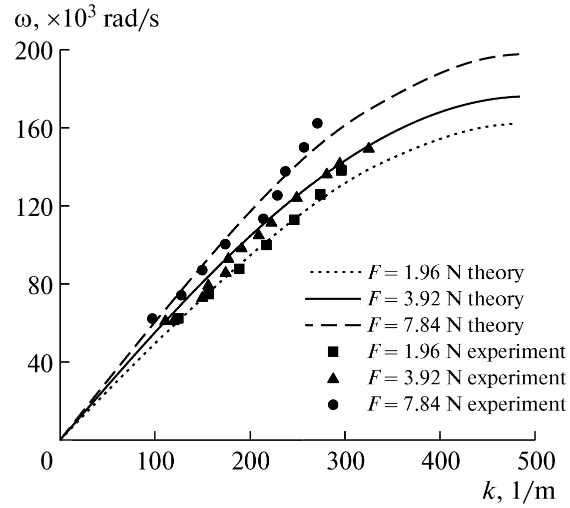

Fig. 5. Dispersion curves of the elastic wave in the chain for three different values of the compression force.

Substituting $h = h_m\sin(\omega t)$, where h_m is the acoustic wave amplitude, in Eq. (20) and applying the time averaging procedure, we obtain

$$\langle V_{gr}\rangle = Ch_0^{1/4}\left[1-\frac{3}{64}\left(\frac{h_m}{h_0}\right)^2\right]. \quad (21)$$

From Eq. (21), it follows that the group velocity of an elastic wave in the chain depends on the constant compression value h_0 and is proportional to the square of the acoustic wave amplitude h_m. For $h_m \longrightarrow 0$, $V_{gr}(h_m \longrightarrow 0) = Ch_0^{1/4}$. Taking $\Delta V_{gr}(h_m) = V_{gr}(h_m \longrightarrow 0) - \langle V_{gr}\rangle$, we obtain

$$\frac{\Delta V_{gr}(h_m)}{V_{gr}(h_m \longrightarrow 0)} = \frac{3}{64}\left(\frac{h_m}{h_0}\right)^2.$$

In the experiment, we measured the propagation time for the maximum of the envelope of the first transmitted pulse. As the amplitude at the radiating transducer increased, the maximum shifted leftward, i.e., the velocity increased. The results of measuring the relative velocity variation on the voltage amplitude A at the radiating transducer are shown in Fig. 6 for four different values of the compression force. The values of the external static force F were as follows: 0.98, 1.96, 3.92, and 7.84 N. For weak compression $F =$ 0.98 N, the dependence was quadratic: $\Delta V/V = 0.0016A^2 - 0.0143A$. The ratio $\Delta V/V$ increased up to

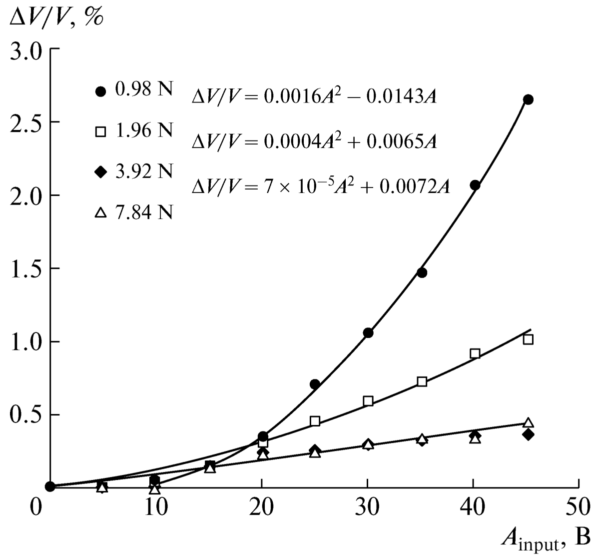

Fig. 6. Dependence of the relative velocity of the elastic wave on the wave amplitude for different values of the static force.

2.7% when the signal amplitude A at the radiating transducer increased to 45 V. The quadratic dependence of the velocity on the voltage points to the presence of a considerable cubic nonlinearity in the one-dimensional chain of balls.

As the compression force increased, the dependence of the velocity on the wave amplitude became weaker: $\Delta V/V = 0.0004A^2 + 0.0065A$; beginning with a force of ~4 N, the dependence became approximately linear and independent of the further increase in the compression force: $\Delta V/V = 7 \times 10A^2 + 0.0072A \approx 0.0072A$. This experimental observation confirms the predicted decrease in the contact nonlinearity with increasing static force.

CONCLUSIONS

Experimental studies of the characteristic features of small-amplitude elastic wave propagation in a one-dimensional model of a periodic unconsolidated structure with Hertzian nonlinearity were carried out. The dependence of the group velocity of an elastic wave on its frequency was calculated and measured. The dispersion relations were calculated for different values of the compression force. The theoretical and experimental dependences were found to be in good agreement. The dependence of the elastic wave velocity on the wave amplitude was measured for different values of the compression force. A quadratic dependence of the velocity of an elastic wave on its amplitude was observed, which points to the presence of a considerable cubic nonlinearity in the one-dimensional chain of balls. The results of the aforementioned studies suggest a conclusion that the elastic linear and nonlinear properties of the system under consideration may vary under the effect of an external force.

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