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NONLINEAR ACOUSTICS

Propagation of Weak Ultrasonic Pulses in an Intense Low-Frequency Wave Field in a Granite Resonator

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Abstract—Results of an experimental study of nonlinear attenuation and carrier frequency phase delay of weak ultrasonic pulses under the effect of an intense low-frequency wave in a bar resonator made from Karelian granite are presented. The effects observed in the experiment are analytically described in terms of the phenomenological equation of state containing hysteretic and dissipative nonlinearities. A frequency dependence of nonlinearity is revealed, and the effective values of nonlinear parameters of granite are estimated for the frequency range from 150 kHz to 1 MHz.

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The topical problems of modern nonlinear acoustics [1–3] include studies of mechanisms responsible for the anomalously high nonlinearity of the so-called microinhomogeneous [4, 5] (or mesoscopic [6]) media, the determination of the laws governing the nonlinear wave processes in them, and the derivation of nonlinear equations of state for these media. The topicality of the aforementioned problems is the result of the fact that the "classical" five-constant (or nine-constant) elasticity theory intended for describing weakly nonlinear homogeneous solids [7–9] fails to explain the nonlinear wave processes observed in experiments with microinhomogeneous media. On the other hand, no "universal" theory adequately describing nonlinear wave processes in such media has ever been developed. Solving these problems is related to the search for materials with a strong acoustic nonlinearity and to experimental investigation of the amplitude–frequency dependences characterizing nonlinear wave processes in them. Experiments show that the acoustic nonlinearity of microinhomogeneous media is frequency-dependent and that the nonlinearity of such media usually exhibits different behavior in the low-frequency and high-frequency ranges: it is hysteretic in the first case and dissipative in the second case.

In this paper, which continues our previous work [10], we present the results of experimental and theoretical studies of the nonlinear acoustic effects manifesting themselves in the propagation of weak high-frequency pulses in the field of an intense low-frequency pumping standing wave in a bar resonator made from Pitkyarant Karelian granite. We analytically describe the amplitude–frequency dependences observed for the attenuation and the carrier frequency phase delay of these pulses under the effect of the low-frequency wave in terms of the equation of state containing a hysteretic nonlinearity and a dissipative one.

The experiments were carried out using the same bar resonator (with a hard boundary and a soft boundary) as that used in [10]. The resonator was made from Karelian granite; its length was $L = 35$ cm, and its cross-section had the form of a square with a side of 1.6 cm. The measurements were performed at room temperature using the setup described in [11, 13]. For the first three longitudinal modes of the resonator, at a small excitation amplitude at which nonlinear low-frequency effects were not observed, the resonance frequencies F_p ($p = 1, 2, 3$) were as follows: $F_1 \cong 3820$, $F_2 \cong 10220$, and $F_3 \cong 17200$ Hz. The errors in measuring the frequencies and the amplitudes of the low-frequency acoustic wave and the high-frequency pulses were $\pm 5 \times 10^{-1}$ Hz, $\pm 5 \times 10^{-2}$ dB, and $\pm 1.6 \times 10^{-1}$ dB, respectively; the error in measuring the phase delay of a pulse was ±5 ns.

In the experiments, in addition to the resonant low-frequency pumping wave, relatively weak ultrasonic pulses were excited in the bar (on the side of its soft boundary). For the pulses transmitted through the bar, we observed and studied the nonlinear attenuation and the carrier phase delay as functions of the strain amplitude ε_m of the intense low-frequency wave. The duration of the pulses was about $\tau = 60$ μs, their carrier frequency f was within the range from 150 kHz to 1 MHz, and the pulse repetition frequency was 26 Hz. After transmission through the bar, the high-frequency pulses were received by an accelerometer placed at the hard boundary of the bar; then, they were supplied to a digital spectrum-analyzing oscilloscope, where their amplitude $U(\varepsilon_m)$ and carrier phase delay $\Delta\tau(\varepsilon_m)$ were measured. The propagation velocity C of high-frequency pulses in the bar, which was

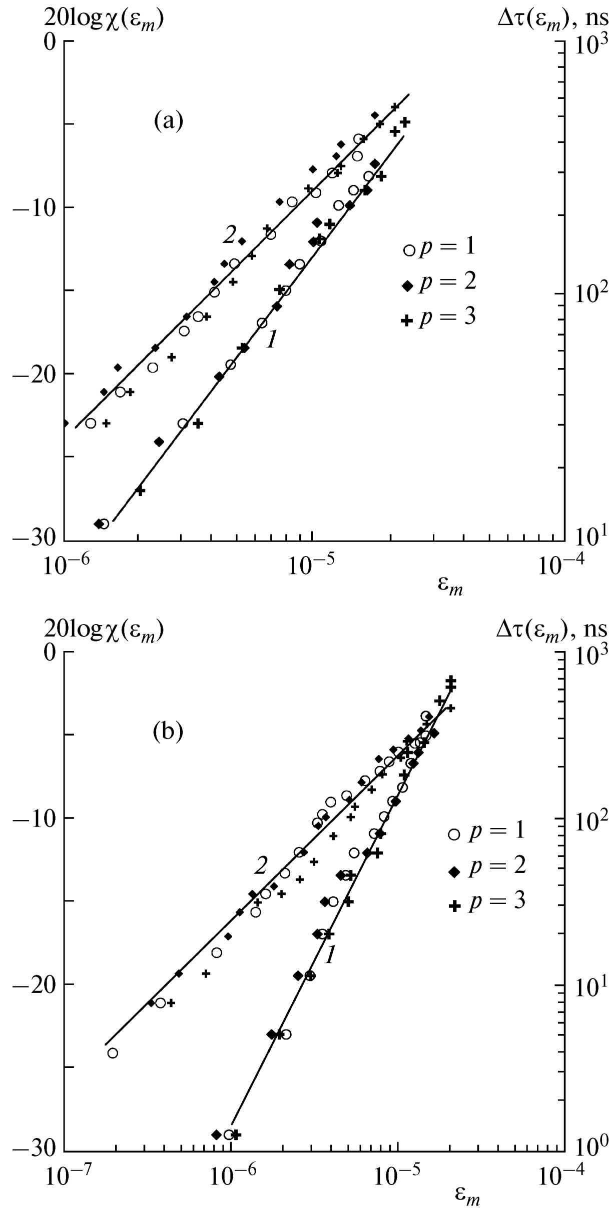

Fig. 1. Dependence of (*1*) the nonlinear attenuation coefficient and (*2*) the carrier frequency phase delay of pulses with the frequencies f = (a) 277 and (b) 490 kHz on the strain amplitude ε_m of the low-frequency wave (at resonance) in the case of the resonator excitation at the first three modes. The straight lines correspond to the dependences $\chi(\varepsilon_m) \sim \varepsilon_m$ and $\Delta\tau(\varepsilon_m) \sim \varepsilon_m$.

determined from the delay (at $\varepsilon_m = 0$), was about 5×10^5 cm/s, so that the pulse length $l = C\tau = 30$ cm was smaller than the length L of the bar and the pulses propagated in the bar in the same way as in free space. As the strain amplitude ε_m of the low-frequency pumping wave increased (in the range $\varepsilon_m > 10^{-6} > \varepsilon^*$, i.e., in the second range considered in [10]), the amplitude of the received high-frequency pulses $U(\varepsilon_m)$ noticeably decreased, while their phase delay $\Delta\tau(\varepsilon_m)$ increased. Figure 1 shows the dependences of the nonlinear attenuation coefficient $\chi(\varepsilon_m) = \ln[U_0/U(\varepsilon_m)]$ (where U_0 is the pulse amplitude in the absence of pumping) and the phase delay $\Delta\tau(\varepsilon_m)$ for pulses of frequencies $f = 277$ and 490 kHz on the strain amplitude ε_m of the low-frequency wave (at resonance) in the case of resonator excitation at its first three modes. From Fig. 1, one can see that, for the first three modes of the resonator, the coefficient $\chi(\varepsilon_m)$ and the delay $\Delta\tau(\varepsilon_m)$ are proportional to the strain amplitude ε_m of the low-frequency pumping wave and do not depend on its frequency F_p; i.e., $\chi(\varepsilon_m) \sim \varepsilon_m$ and $\Delta\tau(\varepsilon_m) \sim \varepsilon_m$. Figure 2 shows the dependences of the coefficient $\chi = \chi(\varepsilon_m)$ and the delay $\Delta\tau(\varepsilon_m)$ on the pulse frequency f in the case of the resonator excitation at the second mode ($p = 2$) at $\varepsilon_m = 10^{-5}$. Despite a certain scatter of experimental points irrespective of the measurement error, one can see that an increase in the pulse frequency f is accompanied by an increase in the coefficient $\chi = \chi(\varepsilon_m)$, $\chi(\varepsilon_m) \sim f$, while the delay $\Delta\tau(\varepsilon_m)$ first (between 150 and 300 kHz) noticeably decreases as $\Delta\tau(\varepsilon_m) \sim f^{-1}$ and then (from 300 kHz to 1 MHz) remains approximately constant: $\Delta\tau(\varepsilon_m) \approx$ const. Such dependences of $\chi(\varepsilon_m)$ and $\Delta\tau(\varepsilon_m)$ on the pulse frequency f testify to a dispersion of the nonlinear acoustic properties of granite, which is due to manifestation of nonlinearity relaxation [11–13].

We note that changes in the amplitude and phase of a weak high-frequency pulse in the field of an intense low-frequency wave are possible in a resonator with a hysteretic nonlinearity alone (as a consequence of the variation of the mean propagation velocity of a high-frequency pulse in the field of an intense low-frequency standing wave) [14]. Calculations show that, for a resonator with superposition of an elastic quadratic hysteresis and an inelastic quadratic hysteresis (see Eqs. (2) and (10) in [10]), the nonlinear attenuation coefficient $\chi_h(\varepsilon_m)$ and the nonlinear phase delay $\Delta\tau_h(\varepsilon_m)$ linearly depend on ε_m and are determined by the expressions

$$\chi_h(\varepsilon_m) = \frac{\eta\gamma_0 + (1-\eta)\pi\beta/2}{2\pi}\varepsilon_m, \quad (1)$$

$$\Delta\tau_h(\varepsilon_m) = \frac{\eta\gamma_0 + (1-\eta)\pi\beta/2}{2\pi^2}\frac{\varepsilon_m L}{C}. \quad (2)$$

Here, $\gamma_0 = \gamma_1 - \gamma_2 + \gamma_3 - \gamma_4 + \dfrac{\pi(\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4)}{4}$; γ_{1-4} and β are the nonlinear parameters of the elastic and inelastic hystereses, respectively; and η and $1 - \eta$ are the concentrations of the defects responsible for the manifestations of the elastic and inelastic hystereses, where $\eta \cong 0.38$.

However, it should be taken into account that the nonlinearity of microinhomogeneous media (including rocks) is characterized by a dispersion, i.e., is frequency-dependent, and, as the frequency Ω of an acoustic wave increases (under the condition that $\Omega > \Omega_{def}$, where Ω_{def} is the defect relaxation frequency), their hysteretic nonlinearity decreases [11–13]. The results of our previous experiments [10] showed that, in the low-frequency range (at least, up to 17 kHz), the hysteretic nonlinearity of the given granite specimen is frequency-independent; hence, $\Omega_p < \Omega_{def}$, where p = 1, 2, 3. However, in the case under study, the frequencies $\omega = 2\pi f$ of ultrasonic pulses could exceed Ω_{def}, so that, in the high-frequency range ($\omega \gg \Omega_3$), the hysteretic nonlinearity of granite could be (and, most likely, was) frequency-dependent. The effective values of its parameters in the high-frequency range should be smaller than the corresponding low-frequency parameters determined in [10]. Still, we will estimate the possible values of $\chi_h(\varepsilon_m)$ and $\Delta\tau_h(\varepsilon_m)$ related to the hysteretic nonlinearity of granite [10] under the assumption that this nonlinearity is frequency-independent, i.e., inertialess. Substituting the values of the parameters $\gamma_1 + \gamma_3 = 8.9 \times 10^3$, $\gamma_2 + \gamma_4 = 3.3 \times 10^3$, and $\beta = 4.2 \times 10^3$, which were determined from the results of the first series of low-frequency experiments, in Eqs. (1) and (2), we obtain $\gamma_0 \cong 15.2 \times 10^3$ and $\eta\gamma_0 + (1 - \eta)\pi\beta/2 \cong 10^4$. For $\varepsilon_m = 10^{-5}$, we have $\chi_h(\varepsilon_m) = 1.6 \times 10^{-2}$, which is almost 14 times smaller than the experimental value $\chi(\varepsilon_m) \cong 2.2 \times 10^{-1}$ measured for f = 150 kHz (Fig. 2a). At the same time, the value of the phase delay proves to be $\Delta\tau_h(\varepsilon_m)$ = 350 ns, which roughly agrees with the experimental value $\Delta\tau(\varepsilon_m) \cong$ 500 ns obtained for the same amplitude $\varepsilon_m = 10^{-5}$ and frequency f = 150 kHz (Fig. 2b). From the experimental dependence $\chi = \chi(\varepsilon_m)$ (Fig. 1a), it follows that the granite specimen under study possesses dissipative nonlinearity [14]. From the experimental dependence $\Delta\tau_h = \Delta\tau_h(\varepsilon_m)$ (Fig. 1b), it follows that the nonlinear phase delay observed for the carrier of a high-frequency pulse is related to the manifestation of the relaxing hysteretic nonlinearity (or the reactive nonlinearity of the form $\gamma|\varepsilon|\varepsilon$, which makes no fundamental difference for the phase delay) [14]. An exact analytic description of the effects under consideration with allowance for the relaxation of all the types of nonlinearity observed in the material (the hysteretic, reactive, and dissipative nonlinearities) presents a complicated problem. Therefore, to simplify our calculations, we perform a qualitative analysis in terms of the phenomenological equation of state that contains the hysteretic [10] and dissipative inertialess nonlinearities [14]:

$$\sigma(\varepsilon, \dot{\varepsilon}) = E[\varepsilon - f(\varepsilon, \dot{\varepsilon})] + \alpha\rho[1 + \delta|\varepsilon|]\dot{\varepsilon}. \qquad (3)$$

Here, σ and ε are the longitudinal stress and the longitudinal strain, $f(\varepsilon, \dot{\varepsilon})$ is the hysteretic function of both strain and strain rate $\dot{\varepsilon}$, α is the linear viscosity coefficient, ρ is the density, δ is the dimensionless dissipative nonlinearity parameter, $|f(\varepsilon, \dot{\varepsilon})| \ll |\varepsilon|$, $\delta|\varepsilon| \ll 1$, $\alpha\delta|\dot{\varepsilon}|/C^2 \ll 1$, and $C^2 = E/\rho$. Since, in the high-frequency range, granite exhibits nonlinear relaxation, the parameters α, δ, γ_{1-4}, and β of the hysteretic function $f(\varepsilon, \dot{\varepsilon})$ depend on the frequency ω of the acoustic pulse ω. In this case, the nonlinear attenuation coefficient $\chi(\varepsilon_m)$ of the pulse due to the dissipative nonlinearity $\alpha\rho\delta|\varepsilon|\dot{\varepsilon}$ is determined by the expression [14]

$$\chi(\varepsilon_m) = \frac{2\alpha\delta\varepsilon_m\omega^2 L}{\pi^2 C^3}, \qquad (4)$$

and the phase delay $\Delta\tau_h(\varepsilon_m)$ is determined by Eq. (2).

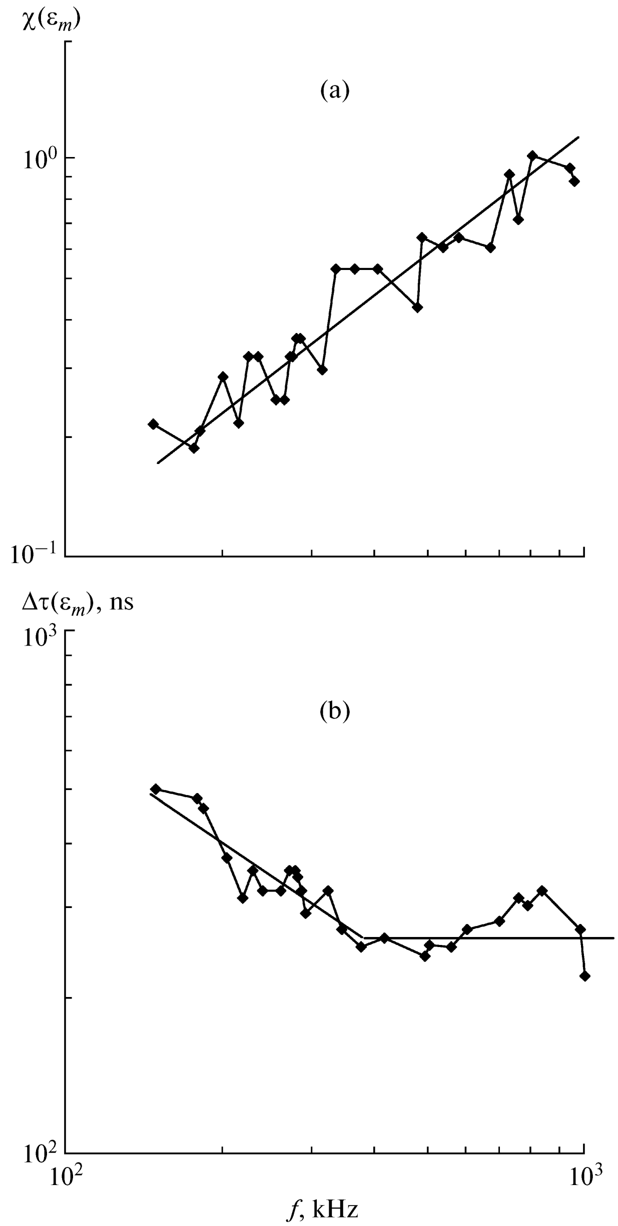

Fig. 2. Dependence of (a) the nonlinear attenuation coefficient and (b) the carrier phase delay on the pulse frequency f in the case of the resonator excitation at the second mode ($p = 2$) at $\varepsilon_m = 10^{-5}$. The straight lines correspond to the dependences $\chi(\varepsilon_m) \sim f$, $\Delta\tau(\varepsilon_m) \sim f^{-1}$, and $\Delta\varepsilon(\varepsilon_m) \approx$ const.

From the comparison of dependences (1)–(4) with the experimental results, we obtain the values of the effective parameters of dissipative and hysteretic nonlinearities of granite for the frequency $f_0 = 150$ kHz ($\alpha\omega_0^2 L/2C^3 \cong 1$): $\delta \cong 6.2 \times 10^4$ and $\eta\gamma_0 + (1 - \eta)\pi\beta/2 \cong 1.4 \times 10^4$. The value of the second parameter is close to the corresponding parameter determined above from the results reported in [10]. From Fig. 2, one can see that, as the frequency f grows (from 150 kHz to 1 MHz), the coefficient $\chi(\varepsilon_m) \sim \alpha(f)\delta(f)f^2$ increases on the average as f/f_0, i.e., $\alpha(f)\delta(f) \sim f^{-1}$; the hysteretic nonlinearity parameter $[\eta\gamma_0(f) + (1 - \eta)\pi\beta(f)/2]$ first (from 150 to 400 kHz) decreases as f_0/f and then (from 400 kHz to 1 MHz) levels off: $\eta\gamma_0 + (1 - \eta)\pi\beta/2 \approx 8 \times 10^3$.

Thus, our experimental studies of nonlinear effects arising in the propagation of weak high-frequency pulses in an intense low-frequency pumping wave field in a Karelian granite bar resonator showed that, in the frequency range between 150 kHz and 1 MHz, the acoustic nonlinearity of granite is anomalously high and frequency-dependent. The nonlinearity contains the dissipative and hysteretic components. As the frequency grows, the dissipative nonlinearity, which is responsible for the attenuation of sound by sound, mainly increases, while the hysteretic nonlinearity, which is responsible for the carrier phase delay of high-frequency pulses, decreases.

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