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CLASSICAL PROBLEMS OF LINEAR ACOUSTICS AND WAVE THEORY

The Contribution of a Lateral Wave in Simulating Low-Frequency Sound Fields in an Irregular Waveguide with a Liquid Bottom

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Abstract—For a two-dimensional irregular waveguide with a bottom in the form of a liquid half-space simulating the coastal zone of a sea shelf, we present calculations on sound propagation at certain low frequencies taking into account the contribution of the integral over the Pekeris branch cut approximated according to the Zavadsky–Krupin technique. Calculations were conducted on the basis of causal matrix equations for the modes obtained in previous studies and that are equivalent to the equations of the cross section method. It is shown how, with a lowering of sound frequency, there is an increase in the contribution to the full field of the branch-line integral corresponding to a lateral wave. Features of transformation of the first propagating mode are established as the section of the cutoff is passed; in such a situation, we have an idea of the wave pattern of the exposure region of the waveguide beyond this section, where propagating modes are absent. As earlier, we perform a comparative analysis of the curves of losses during propagation, corresponding to the solution of exact equations and the description of approximating one-way propagation when allowing for and ignoring coupling of modes.

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INTRODUCTION

In [1], numerical analysis was conducted of low-frequency sound propagation in an irregular two-dimensional Pekeris waveguide with a hydrology corresponding to the real conditions of the coastal zone of an ocean shelf. Calculations were performed using the solution to a boundary-value problem for equations of the cross section method, which can be reduced to an equivalent evolutionary-type problem [2].

The latter represents the first-order matrix equations for acoustic modes over the horizontal coordinate and considers all wave effects of mode coupling and backscattering. In [1], propagation and transformation of modes of a discrete spectrum were analyzed, which in the case of conducting a Pekeris branch cut on a complex plane of horizontal wave numbers κ includes two families: propagating modes and leaky modes. From the point of view of calculating the field, these families were quite enough for the situations seen in [1], supposing the presence of several propagating modes in the waveguide. However, since for a Pekeris waveguide, the full solution in addition to a discrete spectrum of the wave operator also includes the contribution of the lateral wave, that is, a continuous spectrum κ in the form of the branch-line integral [3], which is of considerable interest in studying such situations when the presence of a continuous spectrum in the solution is important. This paper is devoted to this question, and the urgency of the given study is also dictated by interest in the processes by which the energy of modes is transformed at the water–bottom sediments interface [4, 5].

Note that in the available literature, we do not know of any studies for which an irregular Pekeris waveguide was simulated in a similar statement that takes into consideration the contribution of a continuous spectrum. In the known approaches to numerical solution of mode wave equations for a medium with an irregular water–liquid sediments interface, the half-space of sediments is replaced by the authors with a "false bottom" model [3, 6]; that is, an absorbing layer is introduced with an absolutely reflecting lower boundary and a thickness that multiply exceeds the dimensions of the water layer.

This is an extremely inefficient method, because it leads to a sharp increase in the number of modes subject to consideration to describe the field in a waveguide (in this case, despite the presence of losses in a false bottom, all modes are weakly attenuating, in essence, propagating), especially as they approach the source. As an exception, note [7], in which, on the basis of a double-layer waveguide model, attention was paid in calculations to leaky modes and questions on allowance for the continuous spectrum were at least discussed. Also known are the results of asymptotic analysis of the process of propagating modes passing through the area of the critical section, which are reflected, for example, in [8, 9] (see also references in [9]). We will talk about them briefly below.

BASIC EQUATIONS

First, for convenience we will briefly formulate the equations underlying numerical simulation, which are considered in detail in [1, 2, 10]. If we imagine the fields of acoustic pressure p and horizontal velocity v excited in a two-dimensional waveguide (x, z; x is the horizontal Cartesian coordinate) by a linear source $\delta(x-L)\delta(z-z_0)$ of frequency ω by means of local-mode decomposition of the cross section method,

$$p(x,z) = \sum_l \bar{\varphi}_l(x,z)G_l(x),$$
$$v(x,z) = [i\omega\rho(x,z)]^{-1}\sum_l \bar{\varphi}_l(x,z)g_l(x), \tag{1}$$

then for mode functions $G_m(x)$, $g_m(x)$, $m = 1, 2, \ldots$, the following matrix equations over parameter $L \in (L_0, L)$ corresponding to the position of the right section of the waveguide demarcating the irregular and layered areas of the medium are valid:

$$[G_m(x;L)] = -\hat{G}[\bar{\varphi}_m(L,z_0)],$$
$$[g_m(x;L)] = -\hat{g}[\bar{\varphi}_m(L,z_0)];$$
$$\hat{G}(x;L) = \hat{G}(x;x)\exp\left\{\int_x^L d\eta[[i\hat{\kappa}(\eta) - V^T(\eta)] + i[(\hat{\kappa}(\eta)V(\eta) + V^T(\eta)\hat{\kappa}(\eta) - \hat{\kappa}'(\eta)]\hat{G}(\eta;\eta)]\right\},$$
$$\frac{d}{dL}\hat{G}(L;L) = i[\hat{\kappa}(L)\hat{G}(L;L) + \hat{G}(L;L)\hat{\kappa}(L)] - \hat{G}(L;L)V^T(L) - V(L)\hat{G}(L;L) - E + i\hat{G}(L;L)[(\hat{\kappa}(L)V(L) + V^T(L)\kappa(L) - \hat{\kappa}'(L)]\hat{G}(L;L), \tag{2a, b}$$
$$\hat{G}(L;L)|_{L=L_0} = \hat{\kappa}^{-1}(L_0)/(2i).$$

In (1), (2), the vector of the normalized eigenfunctions $[\bar{\varphi}_m(x,z)]$ with the domain of determination $\mathcal{D} \in (h, H)$ in each x-section satisfies a problem on eigenvalues (EV) for a medium with variable density $\rho(x, z)$ in the general case, and boundary conditions at bottom h and surface H correspond to requirements on the pressure and velocity fields.

Also, $\hat{\kappa}(x) = \{\kappa_m(x)\}$ is the diagonal matrix of local EVs, $\hat{\kappa}'(x)$ is the diagonal matrix of derivatives of EVs, E is a unit matrix, $V = \{V_{ml}(x)\} = \int_{\mathcal{D}} \frac{\bar{\varphi}_m(x,z)}{\rho(x,z)}\frac{\partial\bar{\varphi}_l(x,z)}{\partial x}dz$ is a matrix describing mode coupling, and $V^T(x)$ is the transposed matrix V arising in the equations because of variable density [1]. We consider that the irregular field occupies a part of the space ($L_0 \le x \le L$) outside of which the medium is layered, boundaries L, L_0 are matched [2], and the source is located in a water layer in the right section $x_0 = L$. Equation (2b) describes a backscattering matrix $\hat{R}(L;L) = 2i\hat{\kappa}(L)\hat{G}(L;L) - E$. If backscattering can be neglected, the approximation of one-way propagation (OWP) is valid, the solution for which at $\hat{G}(x;x) = \hat{\kappa}^{-1}(x)/(2i)$ is given by the simplified quadrature (2a). The adiabatic approximation is possible later, assuming matrices $V(x) = \{0\}$, $V^T(x) = \{0\}$.

DISCRETE APPROXIMATION OF THE LATERAL WAVE

Mode approach (1), (2) to the solution of 2-D problems obviously assumes the presence of a discrete spectrum of the wave operator. However, as was noted above, for models of a medium with a bottom in the form of the half-space of sediments, which is what the Pekeris waveguide in particular is, the solution also includes the contribution from the continuous spectrum of values κ in the form of the integral over the branch cut. The branch cut is conducted in a particular way from a branch point owing to the presence in the bottom conditions of the double-valued radical $(k_1^2 - \kappa^2)^{1/2}$, where k_1 is the wave number in liquid sediments. Since we will be considering conducting the branch cut on a complex plane of wave numbers κ according to Pekeris, such that $\mathrm{Re}(k_1^2 - \kappa^2) = 0$, $\mathrm{Im}k \in (0, \infty)$, the sound field will be reduced to the sum of propagating and leaky modes and the branch-line integral. Clearly, in the conditions of an irregular waveguide, direct allowance for the branch-line integral is inconvenient for calculations; a natural way of including it in the above-described approach (1), (2) is preliminary discrete approximation. For this purpose the procedure suggested in [11, 12] for layered waveguides has been used. According to it, the half-space of sediments is replaced by a modeling layer of finite thickness with a complex metric of $h - h_0 = |h - h_0|\exp(i\pi/4)$ and an absolutely rigid lower boundary of h_0. Thus, the spectrum of the operator of the problem becomes purely discrete, and the emerging family of new modes, which for brevity we conveniently term cut modes, corresponds to the integral over the branch cut. Now for a sound field, we have representation (1) with three families of modes, the eigenfunctions (EFs) of which are determined within the domain (H, h_0). It has been shown [12, 13] that

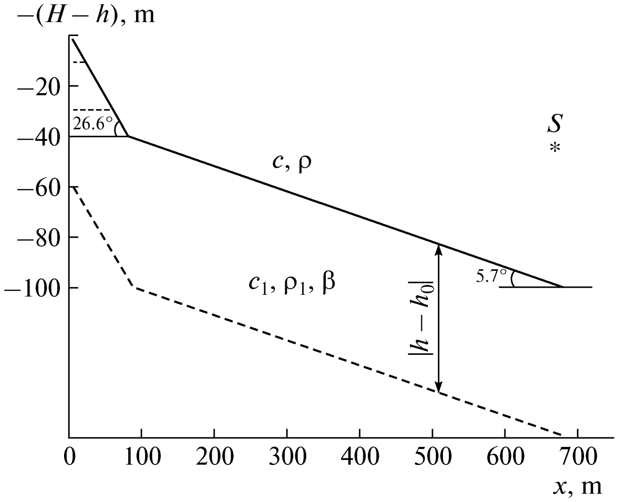

Fig. 1. Model of irregular Pekeris waveguide. Horizontal dotted lines in the upper drawing correspond to the positions of the bottom in [1] at $x < 60$ m for a frequency of 32 Hz and $x < 20$ m for a frequency of 105 Hz. The dashed line shows the introduction of lower boundary h_0 of a layer with a complex metric. S is the arrangement of the source.

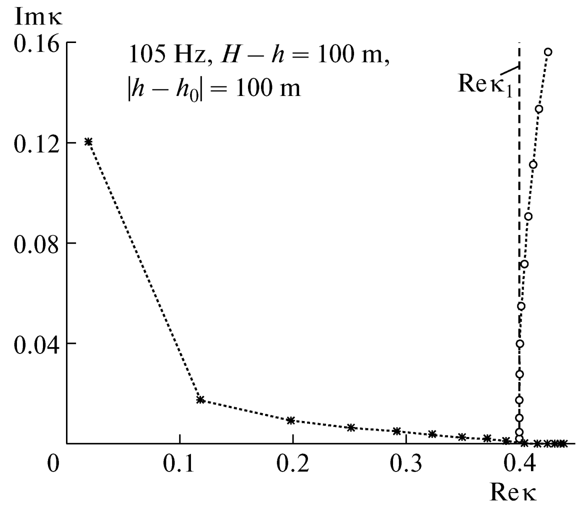

Fig. 2. EV κ_m [m^{-1}] for the model in Fig. 1 in the source section, f = 105 Hz. "*" is κ_m propagating and leaky modes, "o" is κ_m cut modes.

convergence of the given approximation to the integral is uniform, and its accuracy promptly increases $\sim|h - h_0|^{-3}$, slowing down only in the case of cutoff frequencies of the waveguide of $\sim|h - h_0|^{-1}$. As a result, it is possible to restrict calculations to a reasonable volume for different relations of the waveguide parameters and sound frequency of interest. An important circumstance for calculations is that the family of the poles approximating the branch cut and being the EVs of a new family of modes, appears separated in the best way on a complex plane from the EVs of propagating and leaky modes. On the basis of the aforesaid, we will consider propagation of low-frequency sound to a double-layer irregular waveguide with a liquid bottom, explored in [1], only now we will prolong the water–liquid sediments interface $h(x)$ almost to the coastline ($H - h = 1$ m), and we replace the half-space of sediments with a complex layer $h - h_0$ (Fig. 1). We take waveguide parameters similar to [1]: velocities of sound in water, $c = 1500$ m/s; in bottom sediments, $c_1 = 1650$ m/s; the density ratio is $m_\rho = \rho_1/\rho = 2$; uptake in the bottom, $\beta = 0.005$. We will choose acoustic frequencies for simulation of 10, 32, and 105 Hz.

First of all, we will examine the features of the new family of EVs corresponding to the branch cut. As an example, Fig. 2 shows the location on a complex plane of EVs for all families of modes present in the problem at a frequency of 105 Hz in the section of the waveguide with a depth of 100 m. In this case, there are six propagating modes and eight leaky modes with $\mathrm{Re}\kappa_m^2 > 0$ and cut modes.

For demonstration, Fig. 2 shows 12 poles approximating the branch cut at $|h - h_0| = 100$ m lying near line $\mathrm{Re}k_1$. Since the given poles κ_m have an imaginary part increasing with number $\kappa_m'' > 0$, the cut modes corresponding to them are damped when removing from the source, just like leaky modes transferring energy to the bottom (only the law of falling off of a lateral wave more slowly that for leaky modes). This property determined the number of modes approximating the integral over the branch cut necessary during simulation in a layered problem (at a constant depth of the waveguide). For the irregular waveguide, the situation is more difficult, because other factors also play a role. Figure 3 gives a representation of the shape of $\bar{\varphi}_m(z)$ for the cut modes. It is seen that these EFs are significantly less in amplitude than the EFs of usual modes and have a maximum near the bottom which is increasingly expressed with an increase in number. As a result, the lateral wave approximated by the sum of the cut modes sounds mainly the near-bottom area, although with decreasing frequency and reduction in the depth of the waveguide, a large part of the water is highlighted (lower inset to Fig. 3). The thicker the modeling layer $|h - h_0|$, the more frequently the poles of the branch cut will be situated on the curve in Fig. 2, which combines them with each other for visual convenience; however, at the same time, the corresponding EFs $\bar{\varphi}_m(z)$ in Fig. 3 will have a smaller amplitude. Note that in [11–13], generally speaking, there are no restrictions from below on quantity $|h - h_0|$ of the model layer in comparison with the parameters in the problem, except for those that

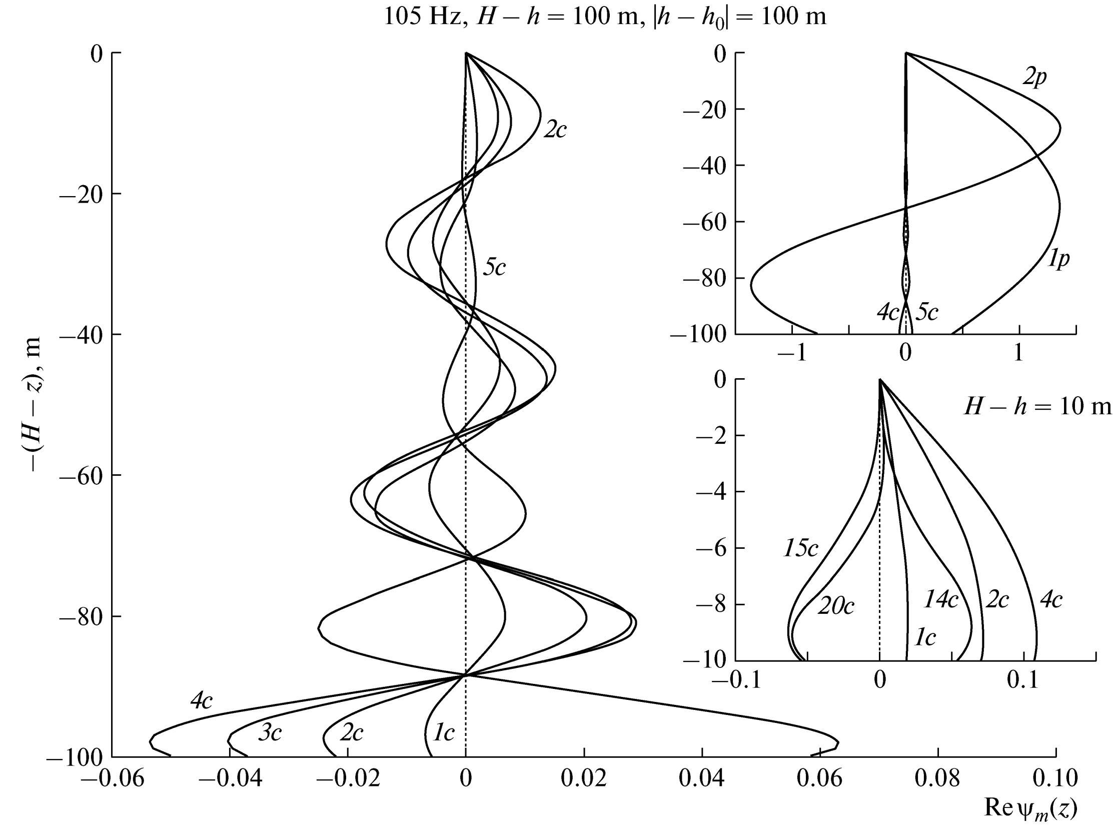

Fig. 3. Dependence on depth of EFs $\psi_m(z) = (H-h)^{1/2}\bar{\varphi}_m(z)$ in the water layer of the model in Fig. 1 in the source section. f = 105 Hz, $|h - h_0|$ = 100 m. *1c...5c*, first five cut EFs. In the upper inset, for comparison, the EFs of the first propagating modes *1p* and *2p* are shown. In the lower inset, the cut EFs at a depth of 10 m.

concern the accuracy in approximating the branch cut given above.

Calculations show, however, that the given parameters should optimally be $|h - h_0| \approx 6$–7 wavelengths of sound λ. In this case, a compromise is reached between the number of necessary cut modes, which should be as small as possible, and the accuracy of the approximations. The main conclusions regarding the contribution of a lateral wave, which can be made with reference to a layered waveguide in accordance with [13] and on the basis of Figs. 2 and 3, are as follows.

In the situation when there are several propagating modes, their field dominates at medial and long-range distances, and near the source, the contribution of leaky modes is also important. The presence of cut modes approximating a continuous spectrum thus does not change the wave pattern and can be disregarded (for example, in the waveguide in Fig. 1 at $f \approx 100$ Hz). Even the presence of only the first propagating mode considerably masks the contribution of the lateral wave; however, in such a situation, it is already noticeable in the transition region from the nearest field to the farthest, and for its correct description, only the first several least attenuating cut modes are necessary.

Finally, the branch cut contribution cannot be neglected in the case when propagating modes are not excited in a waveguide, since only leaky modes do not correctly describe the wave field (for example, Fig. 1 for $f < 10$–20 Hz). As well, there is an increase in the number of cut modes necessary for a correct representation of the contribution from the continuous spectrum; however, in most cases, this number does not exceed 10–15, which is at least ten times less than required for false-bottom models. We will further rely on these conclusions when analyzing sound propagation in an irregular waveguide.

RESULTS OF CALCULATIONS FOR AN IRREGULAR WAVEGUIDE

As follows from quadrature represention (2a), except for the imaginary part $\kappa_m(x)$, the exponent of the exponential curve of the solution will be also described by matrices of mode coupling $V(x)$, $V^T(x)$. For the considered waveguide with constant sound velocity in layers (Fig. 1), in Eqs. (2), it is possible to express the matrix coefficients in an analytical form through a derivative of depth $H(x) = H - h(x)$, the EFs

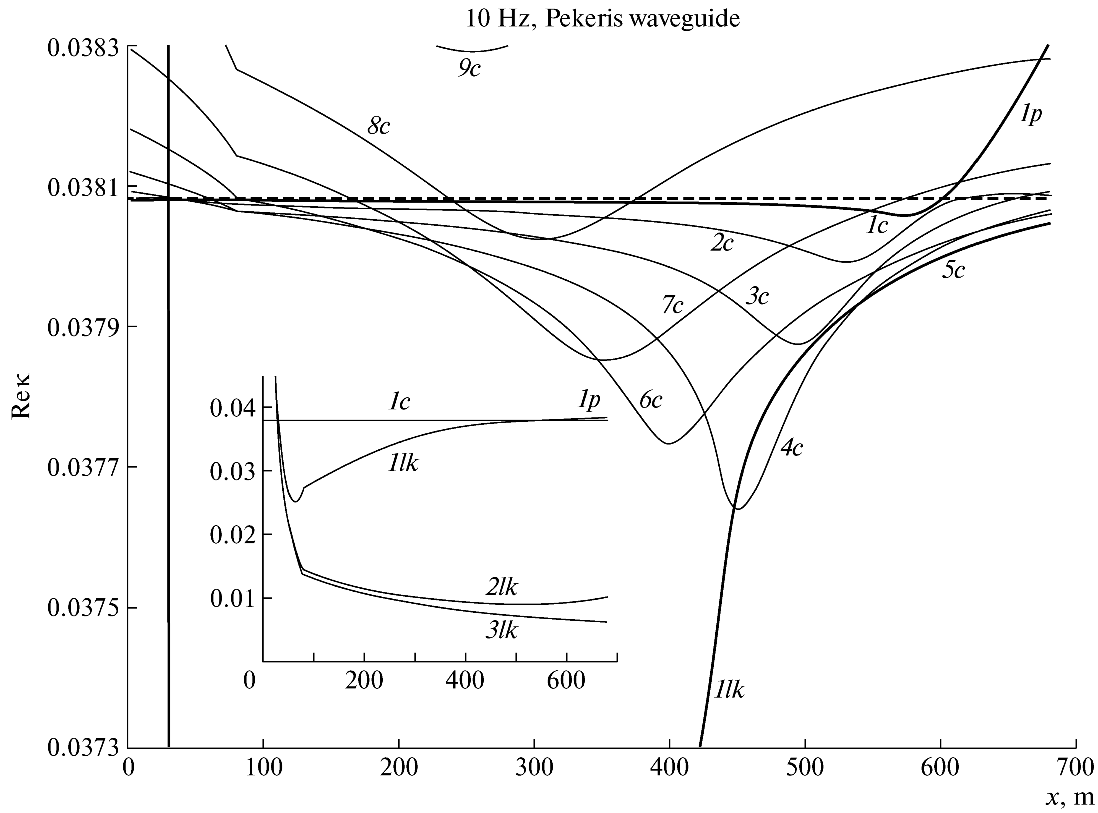

Fig. 4. Dynamics of the real part $\kappa_m(x)$ [m^{-1}] for the model in Fig. 1, $f = 10$ Hz, $|h - h_0| = 1000$ m. *1c*...*9c*, nine cut EVs. *1p* is the EV of the propagating mode, and *1lk*...*3lk* are the EVs of three leaky modes. The dashed line corresponds to Rek_1.

at the bottom $\bar{\varphi}_m(x, h)$ and difference in EVs, so $\{V_{ml}(x)\} \sim H'(x)\bar{\varphi}_m(h(x))\bar{\varphi}_l(h(x))/(\kappa_l^2 - \kappa_m^2)$ [1].

Thus it is clear that the smallness of quantities in numerator at certain x can be compensated by the small difference in the EV of the denominator, which can lead to weakening in the signal attenuation factor with distance for modes in the waveguide, as well as for the first numbers of cut modes. From the physical point of view, this means the possibility of effective redistribution of sound energy between modes of different families. Figures 4 and 5 show the behavior of some EV families of modes in the waveguide in Fig. 1 for a low frequency 10 Hz, well illustrating the expected features of generation of the field. The bold curve of eigenvalue *1p* shows how a propagating mode generated by a source in section $x = 680$ m passing through the critical section at $x \approx 600$ m transforms with a sharp reduction in amplitude (like the example in Fig. 3) to a cut mode (EV *1c*). In the interval of 430 m $< x <$ 600 m, the sound field in a water layer is generated by the cut modes and, to a lesser extent, leaky modes *21k*, *31k*.... At a distance of $x \approx 430$ m, conditions appear for emergence of a leaky mode, and one of the cut modes with substantial growth in amplitude will be transformed to the first leaky mode (EV *1lk*, bold curve). Thus, in the adiabatic consideration by means of cut modes, the continuous transformation of the first (fundamental) mode propagating in the waveguide into a leaky mode beyond the critical section is described, as well as gradual transition of sound energy from the water layer into bottom sediments and its concentration near to interface $h(x)$. The mode transformation process occurs at an interval of $\Delta x \approx 170$ m. Numbers of the cut EVs related to those corresponding for modes *1p*, *1lk* are thus fairly conventional enough, since they are determined by the chosen value $|h - h_0|$ of a modeling layer. From the viewpoint of a more exact theory considering mode coupling in an irregular waveguide, area Δx is also remarkable in its convergence of different EV families, which leads to intensification of intermode interaction and, as a consequence, redistribution of mode energy. So, mode *1p* during transformation to cut mode *1c* will be effective for interacting with cut modes *2c* and *4c*, and leaky mode *1lk* in its appearance is strongly related to profile modes *4c* and *6c*. The result of the given qualitative analysis is supported below by the calculations of Eqs. (1), (2). Figure 6 shows the curves of relative intensity I of the sound field in decibels as functions of distance x for three observation horizons, and Fig. 7 shows the relative amplitudes $p_m = |\bar{\varphi}_m(x, z)G_m(x)/p_0|$ for a series of modes determining the field in a

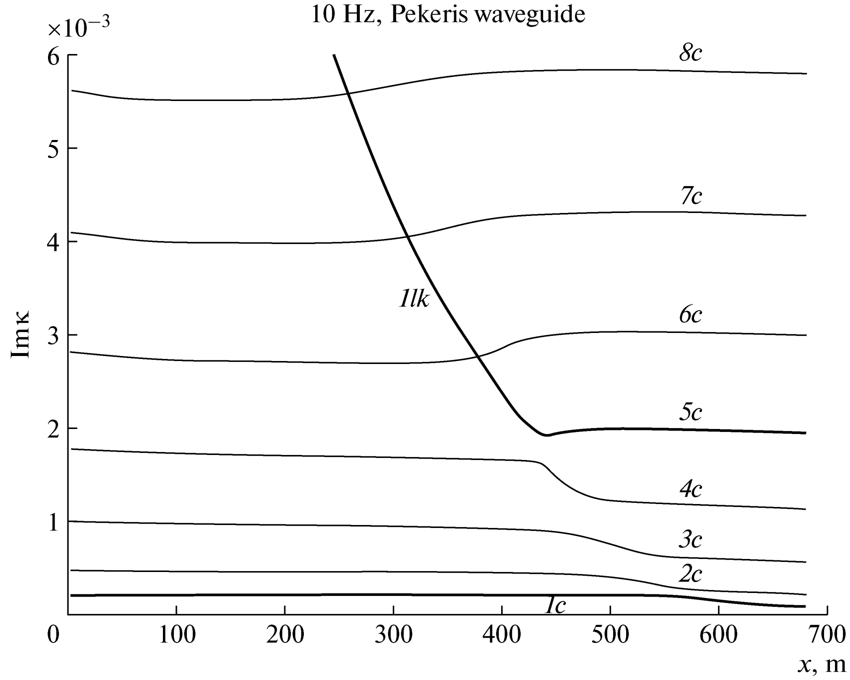

Fig. 5. The same as Fig. 4, the dynamics of the imaginary part $\kappa_m(x)$ [m^{-1}].

waveguide. It is well seen that in comparison to the adiabatic approach when modes do not couple, the propagating mode during departure from a source starts to give up sound energy to cut modes and leaky modes *21k* and *31k*, which intensify and exchange energy with the leaky mode arising at $x \approx 430$ m, which also considerably intensifies. The given mode actually determined the field in a waveguide at $x < 430$ m. Owing to the power exchange between modes, curves of losses for the approximation of one-way propagation (OWP) lie 8–12 dB above the "adiabatic" ones. The OWP theory (thin curves) well describes the fields of modes of pressure and transmission loss (good for intensity!) in a waveguide except for distances $x < 130$ m. Here, the backscattered field reaches 3–4 dB, and the difference in OWP from the exact solution becomes appreciable. To the right of the source, the difference is less than 1 dB. Thus, in the irregular area of a waveguide, the given variation is determined mainly by backscattering of the first leaky modes, and to the right of a source, a single propagating mode. Certainly, the cut modes contribute to backscattering, however, to a very small degree, since the derivative corresponding to EVs $\kappa'_m(x)$ (as well as for a propagating mode [10]) is small; in addition, in full field, this is imperceptible because of the relative smallness in amplitudes of the given modes. For this reason, in [1], in studying backscattering in a two-dimensional Pekeris waveguide, only propagating and leaky modes were considered.

These laws are observed for even an higher frequency of 32 Hz. In this case, it is seen from Fig. 8 that the interval Δx of essential transformation of modes and the intermode power exchange is shifted to the coastline. The cutoff section of the first mode is located now at $x \approx 57$ m, and the leaky mode arises at $x \approx 47$ m, that is, $\Delta x \approx 10$ m, having decreased almost 20 times in comparison to $f = 10$ Hz. These features are observed in the illustration of intensity losses in Fig. 9 in the form of sharp, to 30 dB (the left insert to Fig. 9), drops in the curves of the adiabatic theory compared with other dependences. On even larger parts of the irregular waveguide, exact and approximate dependences are in quite good agreement, which was established and discussed in [1]. The right inset to Fig. 9 presents the dashed curve from this work for a depth of 10 m, constructed by summation of propagating and leaky modes for the hydrology in Fig. 1 with the bottom shown by the dotted line at $x < 60$ m. Apparently, the presence of cut modes is observed not only in the area of substantial redistribution of mode energy, but also in those areas of a waveguide where interference minimums of intensity form, which within several decibels can be both smoothed by presence of a lateral wave (for example, in the areas of $x \approx$ 285, 370, 490 m) and to be accentuated ($x \approx 445$).

With further increase in frequency the new effects are not observed. So, for 105 Hz, the first propagating mode transforms to a leaky by means of cut modes at $x \approx 17.5$ m at an interval $\Delta x \approx 3$ m. Only in this area of the waveguide near the coastline $x < 18$ m presence of a lateral wave will be appreciable. Note that in an

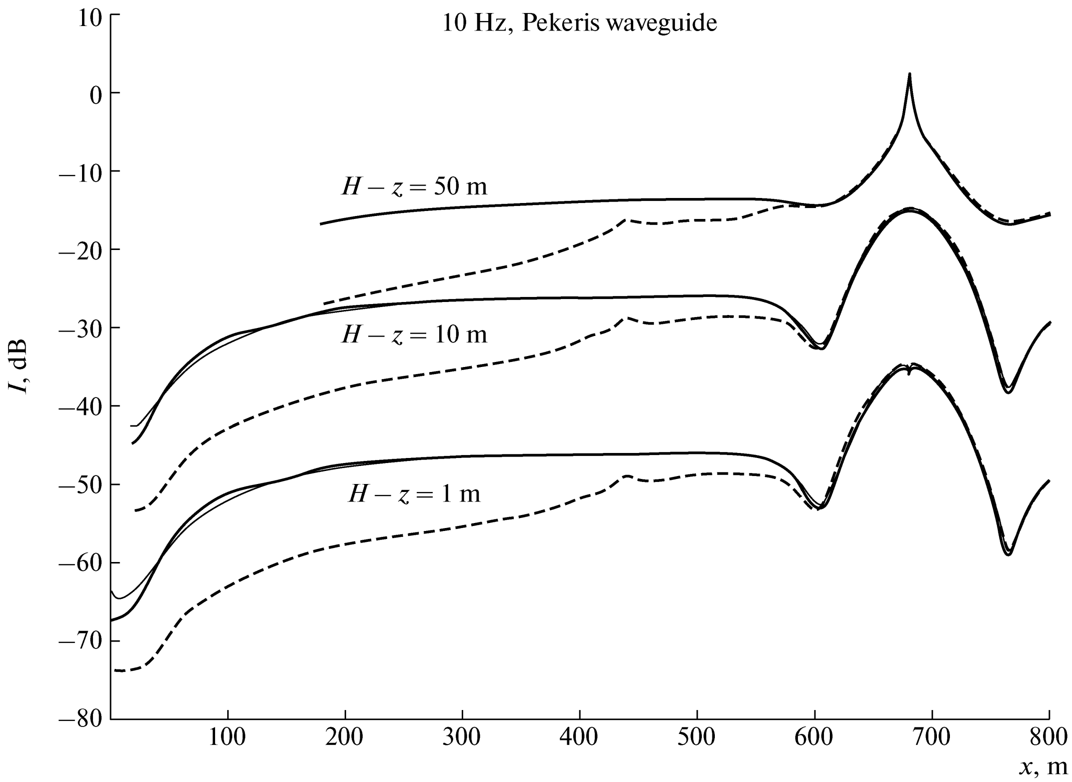

Fig. 6. Curves of transmission losses for the model in Fig. 1. $f = 10$ Hz. Level of curves is referenced to the value of pressure field p_0 in the free medium at a distance of 1 m from the linear source ($p_0 = iH_0^{(1)}(k)/4$). Observation horizons are designated in the figure. A source at a depth of 50 m in section $x_0 = 680$ m. Bold curves, exact solution (1), (2); thin solid curves, OWP; dashed curves, adiabatic theory. $|h - h_0| = 1000$ m.

irregular waveguide, transition of second and higher numbers of propagating modes into corresponding leaky (the opposite is also valid) occurs continuously within the limits of calculation accuracy, "mediation" of cut modes is not required for this. In such sections, there is only convergence of EVs of different families, leading to more intensive intermode power exchange.

Here it is pertinent to briefly compare the sounded results to those of the aforementioned studies [8, 9], in which within the limits of the adiabatic approach and asymptotic analysis, the transition process of a propagating mode through the critical section of a waveguide is described. In the given studies, the concept of leaky modes and (integral) cut modes is not involved, and by means of Airy functions and their derivatives, continuation of a field of a propagating mode for the area of the critical section is carried out. On this basis, a qualitative deduction is made about strong coupling of this mode with continuous spectrum waves (in [9], it is actually only a question of the contribution of a branching point) in the neighborhood of the critical section and the departure of its energy into the bottom (see also [4, 5]). The above illustrations of calculations testify to the fact that at low frequencies (10 Hz, Figs. 6, 7) from the quantitative point of view taking into account the leaky modes and cut modes, even the adiabatic dependences do not drop off rapidly beyond the area of the critical section, and transition of sound energy into bottom sediments is characterized by sufficient graduality. Even more significantly, this process is slowed down within the limits of the more exact theory considering mode coupling. In the latter case, rapid falling off of curves it is not observed even for higher frequencies (32 Hz, Fig. 9).

In summary, we point out that to obtain the dependences presented in Figs. 6, 7, 9, Eqs. (2) for 10–15 cut modes and 4–5 propagating and leaky modes depending on frequency were numerically solved. In addition, in obtaining field (1), we took into account 35 highest leaky modes in the adiabatic approximation for the description of intensity in the immediate neighborhood of the source.

CONCLUSIONS AND DISCUSSION

In the present study, we suggest the results of calculations on propagation of low-frequency sound in an irregular double-layer waveguide with homogeneous

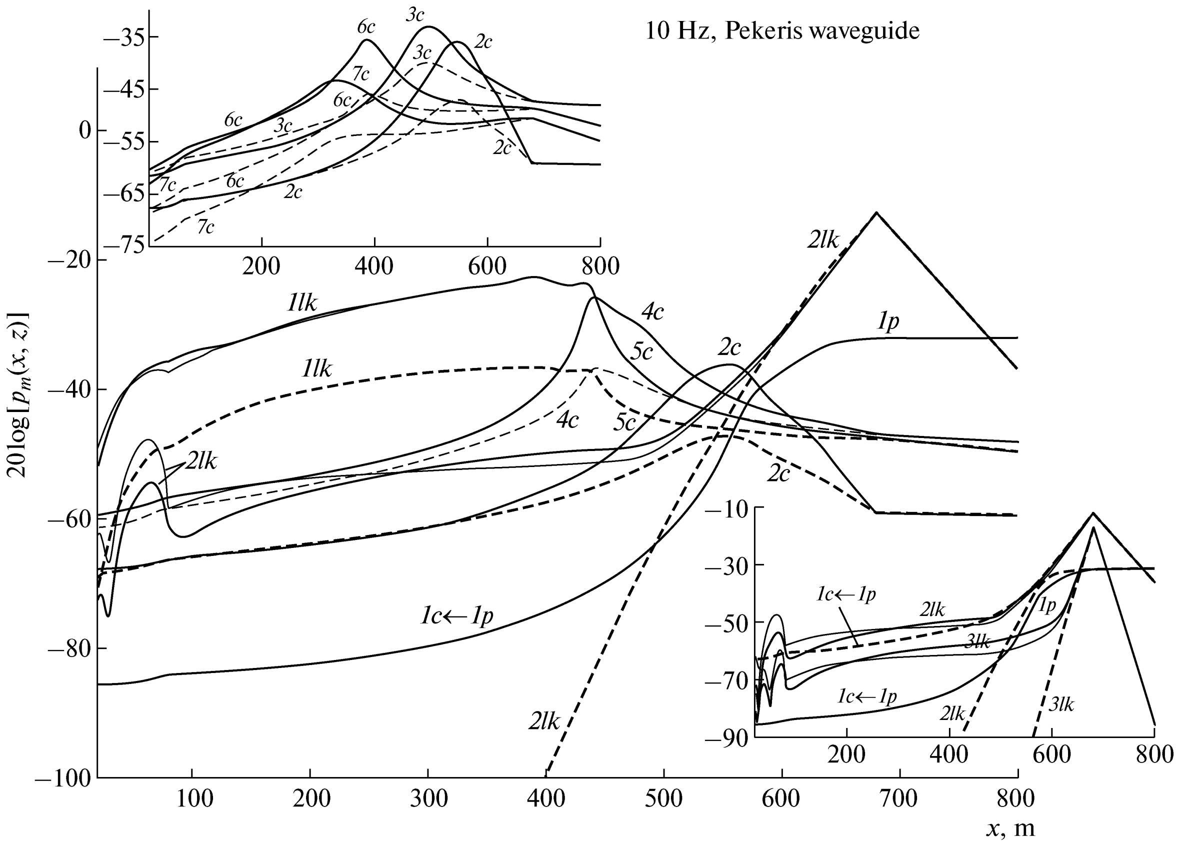

Fig. 7. Dependences on distance in decibels for absolute values of individual modes, normalized to quantity $p_0 = iH_0^{(1)}(k)/4$ for horizon $H - z = 10$ m. $f = 10$ Hz. Bold curves, the exact solution (1), (2); thin solid curves, OWP approximation; dashed curves, adiabatic theory. Upper inset, cut modes *3c*, *6c*, *7c*. Lower inset, leaky modes *2lk*, *3lk* compared with the mode *1p* transforming to the cut mode *1p*. $|h - h_0| = 1000$ m.

water and a liquid bottom taking into account the contribution of the Perekis branch-line integral, which corresponds to a continuous spectrum of the wave operator, which is involved in the problem. The given integral has been approximated by a discrete family of modes according to the procedure suggested by V. Zavadskii and V. Krupin in the case of a layered medium, after which simulation is conducted on the basis of solving the earlier evolutionary matrix equations for the modes. These equations are equivalent to a boundary-value problem in the horizontal direction, posed within the limits of the cross section method. It is shown that when, in an irregular waveguide, there is a cutoff of the first propagating (fundamental) mode, its energy does not "disappear," but passes to cut modes approximating the contribution of a continuous spectrum. These modes describe a lateral wave that propagates primarily in the near-bottom area, reemitting part of the sound energy into bottom sediments via a leaky mode arising in some section. For all intents and purposes, the sound field in the range of distances D between the critical section of the fundamental mode and the coastline is generated by the given leaky mode and the first slowly attenuating cut modes. Owing to the strong mode coupling in the neighborhood of the critical section, this field is not small or rapidly falling, as the adiabatic theory supplemented by asymptotics predicts [8, 9]. In actuality, sufficiently slow de-excitation of sound energy occurs from the waveguide into bottom sediments. Although in the coastal zone of a sea shelf, for any frequency, there is a critical section cutting the fundamental mode, the given effect of redistribution of sound energy between modes of different families is the most substantial in a frequency range lower than 10–20 Hz. In this case, the mentioned interval of distances D reaches hundreds of meters. At frequencies of 100 Hz and above, area D narrows to tens of meters, hugging the coastline. In this case, the presence of a continuous spectrum in the problem manifests itself only in a narrow range of distances, smaller than several metres. Thus, the conclusion is confirmed about the possibility of neglecting the contribution of the integral over the Perekis branch cut in a water layer at frequencies of $f \gtrsim 100$ Hz [13] if the small part of the waveguide near the coastline (beyond the critical section) is of no interest. Note that from the viewpoint of allowance for backscattering, the law is the opposite. So in [1] it is

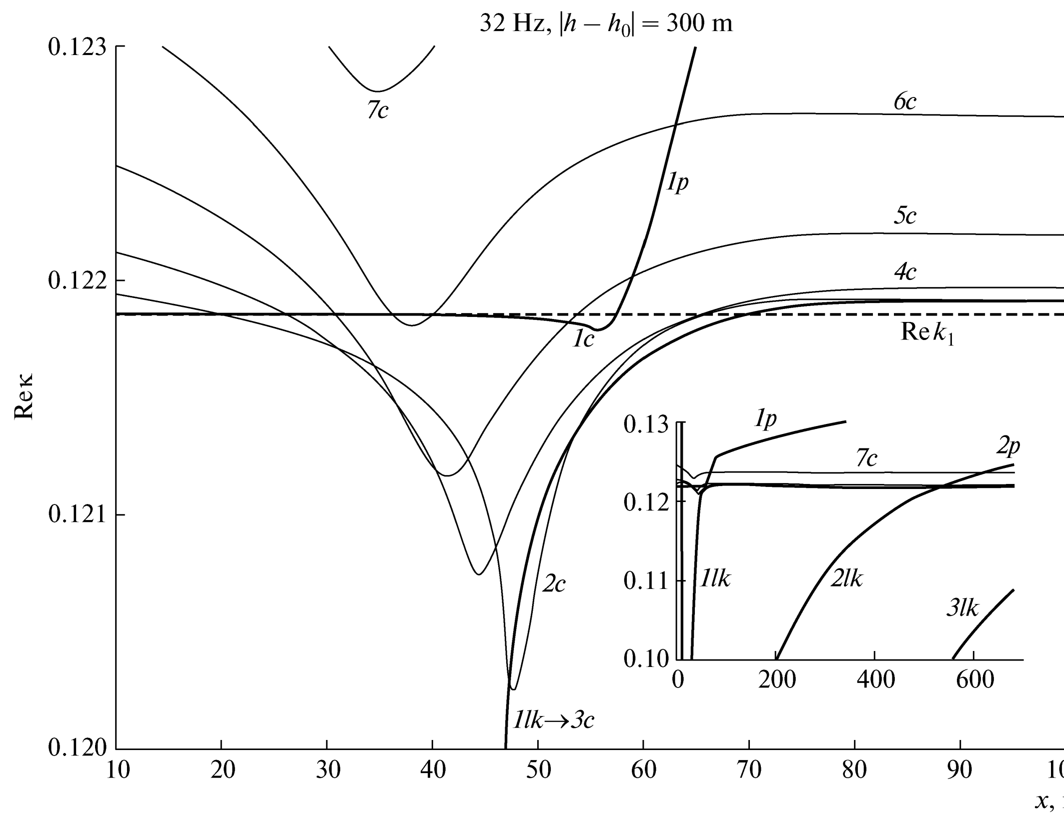

Fig. 8. Dynamics of EVs $\kappa_m(x)$ [m^{-1}] for the model in Fig. 1, $f = 32$ Hz, $|h - h_0| = 300$ m. *1c*...*7c*, the first seven cut EVs. *1p* is the EV of the first propagating mode; *1lk*...*3lk* are the EVs of the three leaky modes.

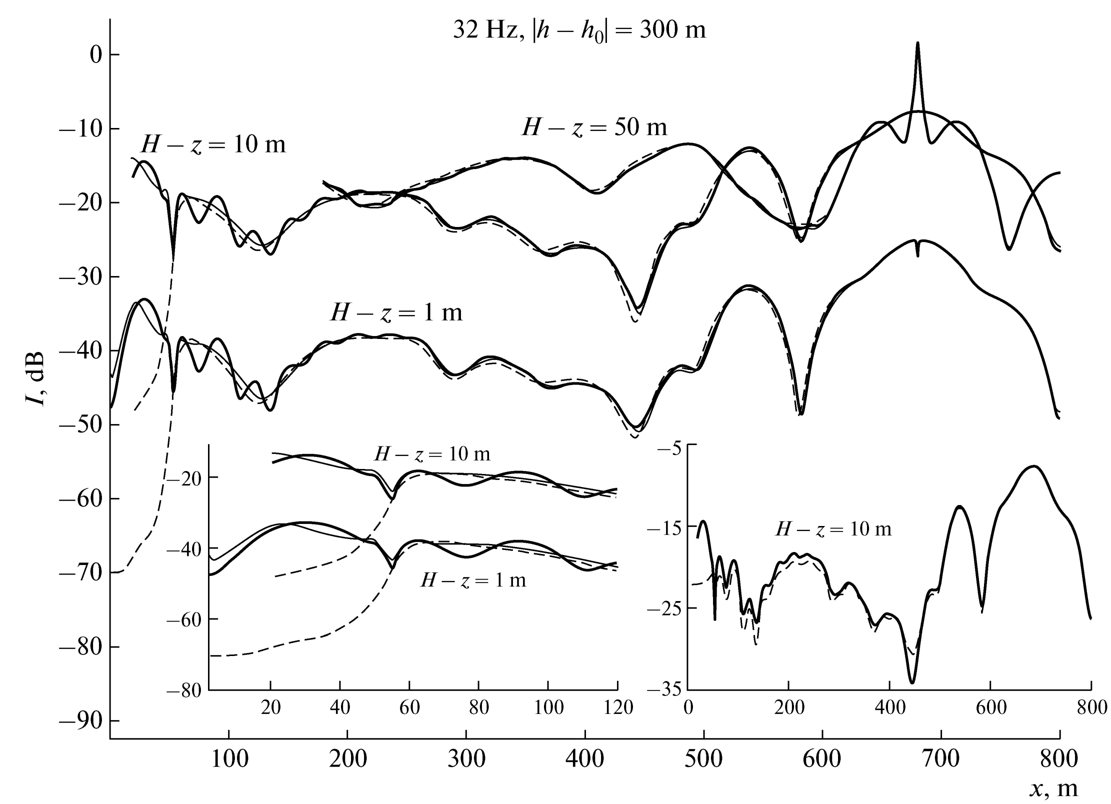

Fig. 9. Curves of transmission losses for the model in Fig. 1. $f = 32$ Hz, $|h - h_0| = 300$ m; the rest, the same as in Fig. 6.

shown that the contribution of a backscattered field increases with increasing frequency to 100 Hz, whereas for 10 Hz it does not exceed 3–4 dB (in pressure intensity) and for the waveguide in Fig. 1, the contribution is localized in the area with a steeper bottom incline.

It is noteworthy that in the literature, one more method of approximating the Perekis branch-line integral is known. It is suggested in [14] for describing broadband signals in a layered waveguide, and it assumes the introduction of a linear refraction index with a complex gradient in the half-space of liquid sediments. As well, instead of a branch cut, there arises a certain discrete family of EVs. The given method in solving the matrix equations suggests dealing with multiple calculation in the coefficients of combinations of Airy functions and their derivatives. In the procedure [11–13], which is used in our study, only elementary functions figure in the coefficients, but in addition, sufficiently fast convergence of similar discretization to the contribution from the branch-line integral is proved. The comparative simplicity of this procedure in respect to the half-space approximation makes it possible to use it for studying irregular waveguides with more complicated bottom models taking into account the stratification of parameters and elastic properties in the upper sediment layers.

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