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Electroacoustic Waves Confined by a Moving Domain Wall Superlattice of a Ferroelectric Crystal

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Abstract—The dispersion properties of electroacoustic wave modes confined by a superlattice of uniformly moving 180° domain walls in a tetragonal ferroelectric crystal are considered. It is shown that the manifold of partial electroacoustic interfacial waves in the lattice is restricted to the first allowed band, the configuration of which in the plane of spectral variables can significantly vary under the action of the moving domain walls.

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INTRODUCTION

The spectral properties of electroacoustic interfacial waves (EIWs) in ferroelectrics with a static periodic domain structure forming a superlattice have been widely investigated [1, 2]. It has been established [3, 4] that, due to the interaction between electroacoustic oscillations of adjacent domain walls (DWs), the phonon spectra of a ferroelectric with the periodic domain structure can strongly differ from the oscillation spectrum of an ordinary single-domain sample. This results in the formation of band gaps in the region of existence of the collective oscillations over the entire lattice volume and strong modification of such EIW spectral parameters as interfacial localization and phase velocity.

The existing methods of obtaining structures with different lattice periods and the possibility of controlling the period [5, 6] open up possibilities for application of the domain superlattices of ferroelectrics as sound conductors for various devices of acoustodomain electronics [7]. Additional opportunities, such as EIW crystal scanning or Doppler transformation of a frequency spectrum, should be attributed to DW moving. Therefore, it is interesting to extend the results of investigation of the acoustic properties of the static superlattices of ferroelectric crystals [1, 2] to the case of dynamic superlattices, the specific feature of which is controlled displacement of the confining domain walls. Below, we consider EIW propagation of equidistant uniformly moving 180° DWs in the dynamic superlattice using the example of 4-mm ferroelectric crystals. Preliminary results were briefly reported in [8]. Unfortunately, in the aforementioned study, given analytical representation of the dispersion relation for the EIWs by a quadratic equation in the sum of the quantities forming a free term, a numerical factor 4 prior to one of the terms was missed. The error did not affect the numerical results and, as a matter of fact, did not reflect on the qualitative conclusions on the changes in the EIW spectra caused by DW motion. Nevertheless, it is necessary to correct the analytical expressions for the EIW spectra, which was an additional stimulus for this study.

STATEMENT OF THE PROBLEM AND FUNDAMENTAL EQUATIONS

The geometry of the problem is presented in Fig. 1, where the dynamic superlattice of equidistant 180° DWs with 010) orientation is shown in the accompanying frame of reference $\tilde{x}0\tilde{y}\tilde{z}$. By virtue of the inequality $V_D \ll c$, where c is the speed of light and V_D is the velocity of DW motion, this frame is related to the laboratory one $x0yz$ via the Galilean transformation

$$\tilde{x} = x, \quad \tilde{y} = y - V_D t, \quad \tilde{z} = z, \quad \tilde{t} = t, \qquad (1)$$

where t is time. Crystal symmetry, type of polarization of waves, and geometry of their propagation are taken as the same as in [1, 2], with the sole correction that the consideration is limited to the modes localized on DWs of the superlattice. Until now, the localized EIW modes have been considered only at small numbers of moving DWs, including the cases of a single moving DW [9], a moving stripe domain (i.e., a pair of DWs) [10], and pairs of moving adjacent stripe domains of different thicknesses (a trio of DWs) [11].

As the main structure-forming element of the superlattice, we chose a pair of adjacent stripe domains of a ferroelectric with the interfaces $\tilde{y} = 0$, d, and $2d$ shown in Fig. 1 in dark color. The remaining domains are formed by translation of this pair to whole number n of lattice periods $2d$ in the positive ($n > 0$) or negative ($n < 0$) direction of the $\tilde{y}$ axis. We ascribe the number $j = 1$ to the lower domain of the pair of a lattice unit cell at $0 < \tilde{y} < d$ and the number $j = 2$ to the upper domain at $d < \tilde{y} < 2d$. Thus, each domain of the lattice will be defined by the translation number $n = 0, \pm 1, \pm 2, \pm 3, \ldots$ and the pair number $j = 1$ or 2; the value $n = 0$ corresponds to the unit (initial) cell.

In view of the aforesaid, under the conditions of specified DW motion at the velocity $V_D \parallel y \parallel [010]$ the current coordinates in the laboratory frame are $y_n = V_D t + nd$. Considering that antiparallel polarizations in domains of the lattice unit cell are related to the sign alteration of piezoelectric modulus e_{1s}, which is the only active one under the given conditions, we assume

$$e_{15}^{(j)} = (-1)^{j+1} e, \quad e > 0. \qquad (2)$$

Extend the validity of this condition over the entire lattice, assuming that in Eq. (2) we have

$$j = \begin{cases} 1 \text{ for } nd + V_D t < y < (n+1)d + V_D t, \\ 2 \text{ for } (n+1)d + V_D t < y < (n+2)d + V_D t. \end{cases} \qquad (3)$$

The lattice cell fields are described by the solutions of the equations [9–11]

$$\frac{\partial^2 u_j}{\partial t^2} = c_t^2 \nabla^2 u_j, \quad \nabla^2 \Phi_j = 0, \qquad (4)$$

where c_t is the velocity of shear wave propagation in a single-domain sample and Φ_j is the portion of the total electric potentials in domains

$$\varphi_j = \frac{4\pi e_{1.5}^{(j)}}{\varepsilon} u_j + \Phi_j \qquad (5)$$

describing the near-interface electric fields induced by piezoelectric polarization charges from DWs. In virtue of Eq. (1), Eqs. (4) and (5) will take the following form in the accompanying frame of reference:

$$\left(\frac{\partial}{\partial \tilde{t}} - V_D \frac{\partial}{\partial \tilde{y}}\right)^2 u_j = c_t^2 \tilde{\nabla}^2 u_j, \quad \tilde{\nabla}^2 \Phi_j = 0. \qquad (6)$$

Then, assuming that the waves propagate in the plane $\tilde{x}0\tilde{Y}$ in the positive direction of the $\tilde{x}$ axis, with regard for proportional displacements u_j and potentials Φ_j with the exponential coefficient $\exp[i(k_\parallel \tilde{x} - \Omega \tilde{t})]$, we conclude that the EIWs are noncollinear: $\mathbf{k} = (k_\parallel, k_\perp, 0)$, $k_\perp \neq 0$ (Fig. 1). Deviation of the EIW front from the orthogonal position is determined by the transverse

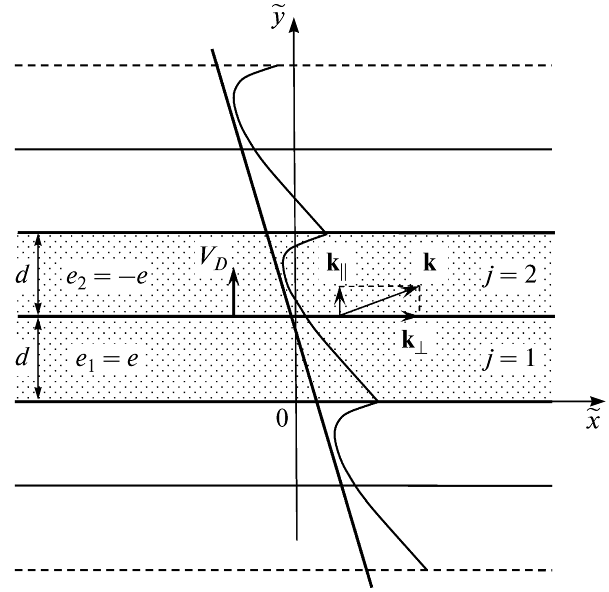

Fig. 1. Dynamic superlattice of uniformly moving equidistant 180° DWs in the accompanying frame of reference.

component of wave vector $k_\perp$ [8]:

$$k_\perp = \frac{\Omega}{c_t} \frac{\beta}{1 - \beta^2}, \qquad (7)$$

where $\beta = V_D / c_t$.

Assume also that the EIW length is much less than the characteristic size of a crystal. Under such conditions, the interfacial effects at the external interfaces of a ferroelectric, as well as the shape of the latter, will not have a considerable effect on the behavior of the electroacoustic waves and can be disregarded.

DERIVATION OF THE EIW DISPERSION EQUATION FOR THE DYNAMIC LATTICE OF A FERROELECTRIC CRYSTAL

Solutions of Eqs. (6) within the pair of domains numbered with the indices $j = 1$ and 2 of the unit cell can be presented as

$$\begin{aligned} u_1(\tilde{y}) &= [A_1 \exp(-s\tilde{y}) + A_2 \exp(s\tilde{y})] \\ &\times \exp(ik_\perp \tilde{y}) \exp[i(k_\parallel \tilde{x} - \Omega \tilde{t})], \\ \Phi_1 &= [C_1 \exp(k_\parallel \tilde{y}) + C_2 \exp(-k_\parallel \tilde{y})] \\ &\times \exp[i(k_\parallel \tilde{x} - \Omega \tilde{t})], \quad (0 < \tilde{y} < d) \end{aligned} \qquad (8)$$

and

$$\begin{aligned} u_2(\tilde{y}) &= [B_1 \exp(-s\tilde{y}) + B_2 \exp(s\tilde{y})] \\ &\times \exp(ik_\perp \tilde{y}) \exp[i(k_\parallel \tilde{x} - \Omega \tilde{t})], \\ \Phi_2 &= [D_1 \exp(k_\parallel \tilde{y}) + D_2 \exp(-k_\parallel \tilde{y})] \end{aligned} \qquad (9)$$

$$\times \exp[i(k_{\parallel}\tilde{x} - \Omega\tilde{t})], \quad (d < \tilde{y} < 2d),$$

where $\Omega = \omega(1 - \beta^2)$ is the EIW frequency in the accompanying frame of reference and s is the coefficient of interfacial localization

$$s = \frac{1}{1-\beta^2}\sqrt{k_{\parallel}^2(1-\beta^2) - \frac{\Omega^2}{c_t^2}}$$

related to frequency ω in the laboratory frame by the formula $\omega = c_t\sqrt{k_{\parallel}^2(1-\beta^2)^{-1} - s^2}$. To sew the domain fields at the internal interface $\tilde{y} = d$, we will use the standard conditions of continuity of shear displacements, potentials, shear components of the stress tensor $T_{yz}^{(j)}$, and y-components of the electric inductions [12]:

$$\varphi_1(\tilde{y})|_{\tilde{y}=d} = \varphi_2(\tilde{y})|_{\tilde{y}=d}, \quad u_1|_{\tilde{y}=d} = u_2|_{\tilde{y}=d},$$
$$\left.\frac{\partial\Phi_1}{\partial\tilde{y}}\right|_{\tilde{y}=d} = \left.\frac{\partial\Phi_2}{\partial\tilde{y}}\right|_{\tilde{y}=d}, \tag{10}$$
$$\left.\left(c_{44}^*\frac{\partial u_1}{\partial\tilde{y}} + e\frac{\partial\Phi_1}{\partial\tilde{y}}\right)\right|_{\tilde{y}=d} = \left.\left(c_{44}^*\frac{\partial u_2}{\partial\tilde{y}} - e\frac{\partial\Phi_2}{\partial\tilde{y}}\right)\right|_{\tilde{y}=d}.$$

Analogous boundary conditions should be written also at the external interfaces of the cell: $\tilde{y} = 0$ and $\tilde{y} = 2d$ ($\tilde{y} = 2nd$ and $\tilde{y} = (n+1)2d$, if a cell of arbitrary number n is assumed). Then, it is necessary to set the requirements of translational invariance of the solution: $u(\tilde{y}) = u(\tilde{y} + 2d)$, $\varphi(\tilde{y}) = \varphi(\tilde{y} + 2d)$, and $\Phi(\tilde{y}) = \Phi(\tilde{y} + 2d)$. The latter are known in the lattice theory as the Bloch cyclic conditions [13], which allow elimination of recurrent connections between wave amplitudes in the neighboring periodic regions that follow from the conditions at the external interfaces. In other words, relying upon the cyclic solution property stated for the extracted lattice unit cell (Fig. 1) in the form of the Bloch theorem [13]

$$u_1(0) = u_2(2d)\exp(i\chi 2d),$$
$$\varphi_1(0) = \varphi_2(2d)\exp(i\chi 2d), \tag{11}$$
$$\text{and } \Phi_1(0) = \Phi_2(2d)\exp(i\chi 2d),$$

where χ is the Bloch wave number, one is allowed not to use the external boundary conditions explicitly.

According to the definition of [12], the transition matrix connects the value of the field (or the fields enchained with one another) at the layer "input" with the value at the "output." For convenience of the further transformations, we make denotation

$$\sigma_j(\tilde{y}) = \frac{\partial\Phi_j}{\partial\tilde{\psi}}, \quad \alpha_j(\tilde{y}) = c_{44}^*\frac{\partial u_j}{\partial\tilde{y}} + e_{1.5}^{(j)}\frac{\partial\Phi_j}{\partial\tilde{y}} \tag{12}$$

and, based on (8) and (9), express amplitude constants A_j, B_j, C_j, and D_j via the field values at the interface $\tilde{y} = 0$ (at $j = 1$) or at the interface $\tilde{y} = d$ (at $j = 2$). Then, using backward substitution of quantities A_j, B_j, C_j, and D_j, with regard to relations (5) and (12), obtain

$$u_j(\tilde{y}) = mu_j + \frac{m_1}{c_{44}^*}\alpha_j + 0\times\varphi_j + \frac{e_{1.5}^{(j)}m_1}{c_{44}^*s}\sigma_j,$$

$$\alpha_j(\tilde{y}) = \left(\frac{c_{44}^*(s^2+k_\perp^2)m_1}{s} - c_{44}^*\mathcal{K}^2k_\parallel\sinh(k_\parallel\tilde{y})\right)u_j$$
$$+ m'\alpha_j + e_{1.5}^{(j)}k_\parallel\sinh(k_\parallel\tilde{y})\varphi_j$$
$$+ e_{1.5}^{(j)}(\cosh(k_\parallel\tilde{y}) - m')\sigma_j,$$

$$\varphi_j(\tilde{y}) = (-1)^{j+1}\frac{c_{44}^*\mathcal{K}^2(m - \cosh(k_\parallel\tilde{y}))}{e_{1.5}^{(j)}}u_j \tag{13}$$
$$+ (-1)^{j+1}\frac{\mathcal{K}^2m_1}{e_{1.5}^{(j)}s}\alpha_j$$
$$+ \cosh(k_\parallel\tilde{y})\varphi_j + \left(\frac{\sinh(k_\parallel\tilde{y})}{k_\parallel} - \frac{\mathcal{K}^2m_1}{s}\right)\sigma_j,$$

$$\sigma_j(\tilde{y}) = (-1)^j\left(\frac{c_{44}^*\mathcal{K}^2k_\parallel}{e_{1.5}^{(j)}}\right)\sinh(k_\parallel\tilde{y})u_j$$
$$+ 0\times\alpha_j + k_\parallel\sinh(k_\parallel\tilde{y})\varphi_j + \cosh(k_\parallel\tilde{y})\sigma_j.$$

In expressions (13), as everywhere above, $\mathcal{K}$ is the coefficient of electromechanical coupling of a single-domain crystal and quantities u_j, α_j, φ_j, and σ_j in the right-hand side of the equalities specify the values of the corresponding fields and their combinations by formulas (12) at $\tilde{y} = 0$ for the index $j = 1$ and at $\tilde{y} = d$ for $j = 2$. Recall also that $c_{44}^* = c_{44} + 4\pi e^2/\varepsilon$, whereas m, m', and m_1 denote the values functionally independent of transverse coordinate $\tilde{y}$:

$$m = \left[\cosh(s\tilde{y}) - \frac{k_\perp}{s}\sinh(s\tilde{y})\right]\exp(-ik_\perp\tilde{y}),$$

$$m' = \left[\cosh(s\tilde{y}) + \frac{k_\perp}{s}\sinh(s\tilde{y})\right]\exp(-ik_\perp\tilde{y}),$$

$$m_1 = \sinh(s\tilde{y})\exp(-ik_\perp\tilde{y}).$$

Hence, by virtue of Eq. (7), in particular at $V_D = 0$, we have $m = \cosh(s\tilde{y})$ and $m_1 = \sinh(s\tilde{y})$.

Expressions (13) represent two (by the number $j = 1$ and 2) systems of four nonuniform algebraic equations that make it possible to associate the fields and their combinations at the interface $\tilde{y} = 0$ ($\tilde{y} = d$) of the first (second) domain with their corresponding values inside these domains. If in (13) the respective values $\tilde{y} = d$ and $\tilde{y} = 2d$ are chosen for the numbers $j = 1$ and $j = 2 - \tilde{y} = 2d$, then the mentioned systems of algebraic equations will acquire the form

$$\begin{pmatrix} u_j(jd) \\ \alpha_j(jd) \\ \varphi_j(jd) \\ \sigma_j(jd) \end{pmatrix} = \mathbf{L}_j \begin{pmatrix} u_j((j-1)d) \\ \alpha_j((j-1)d) \\ \varphi_j((j-1)d) \\ \sigma_j((j-1)d) \end{pmatrix}. \quad (14)$$

Here, $\mathbf{L}_j$ is the 4×4 matrix formed by the coefficients from the right-hand side of Eqs. (14) at $j = 1$ and 2.

In order that Eqs. (14) will acquire the same structure as that in relations (11), i.e., allow expressing the fields at the domain "input" via the fields at the domain "output," we should solve system (14) with respect to the right-hand elements of the column matrix. For this purpose, we introduce the inverse matrix $\mathbf{M}_j = \mathbf{L}_j^{-1}$ called the transition matrix. After elementary transformations based on the expressions for the coefficients from Eqs. (14), we obtain:

$$\mathbf{M}_j = \begin{pmatrix} \frac{m}{\delta} & -\frac{m_1}{c_{44}^*} & 0 & \frac{e_j m_1}{c_{44}^* s\delta} \\ -\frac{c_{44}^*(s^2+k_\perp^2)}{s\delta} m_1 + \mathcal{K}^2 c_{44}^* k_\parallel \sinh(k_\parallel d) & \frac{m'}{\delta} & -e_j \sinh(k_\parallel d) & e_j(-m'/\delta + \cosh(k_\parallel d)) \\ \frac{\mathcal{K}^2 c_{44}^*}{e_j}(m/\delta - \cosh(k_\parallel d)) & \frac{-\mathcal{K}^2 m_1}{e_j s\delta} & \cosh(k_\parallel d) & \frac{-\mathcal{K}^2 m_1}{e_j s\delta} - \frac{\sinh(k_\parallel d)}{k_\parallel} \\ \frac{\mathcal{K}^2 c_{44}^* k_\parallel \sinh(k_\parallel d)}{e_j} & 0 & -k_\parallel \sinh(k_\parallel d) & \cosh(k_\parallel d) \end{pmatrix},$$

where $\delta = \exp(-2ik_\perp d)$.

The product $\mathbf{M} = \mathbf{M}_1 \times \mathbf{M}_2$ of the "domain" transition matrices for the domains forming the lattice unit cell determines the transition matrix on the complete period of the structure by the known technique [13]. The elements of this matrix are cumbersome even for the static lattice and, for this reason, are not presented in explicit form. To facilitate derivation of their explicit representation, simplifying assumptions are usually made: most often, it is the assumption of weak wave coupling in piezoelectrics expressed by the condition $\mathcal{K}^2 \ll 1$. To eliminate this limitation, further transformations during derivation of the dispersion relation for the EIWs were made in a PC symbolic computation mode using the Mathematika package.

Next, we follow a standard procedure of derivation of the dispersion relation for waves in an unbounded lattice [13]. In virtue of cyclic Bloch equations (11), this procedure suggests determination of the eigenvalues $\lambda = \exp(2i\chi d)$ of matrix $\mathbf{M}$. For this purpose, we find and put to zero the determinant $\mathrm{Det}|\mathbf{M} - \lambda\mathbf{E}| = 0$, where $\mathbf{E}$ is the 4×4 unit matrix, and χ is the Bloch wavenumber characterizing the degree of phase synchronism of the electroacoustic oscillations on DWs over the lattice period. After a number of transformations, the dispersion equation for EIWs can be written in the following form:

$$\lambda^4 + \lambda^3 Q(k_\parallel, s) + \lambda^2 P(k_\parallel, s) + \lambda R(k_\parallel, s) + \frac{1}{\delta^2} = 0. \quad (15)$$

At the specified geometric and material parameters, quantities Q, P, and R serving as coefficients in Eq. (15) represent the following functions of spectral variables $k_\parallel$ and s:

$$Q(k_\parallel, s) = -2\cosh(2sd)\delta^{-1} - 2\cosh(2k_\parallel d) + \frac{8\mathcal{K}^2 \sinh(sd)\sinh(k_\parallel d)}{s\delta^{1/2}},$$

$$P(k_\parallel, s) = 1 + \frac{1}{\delta^2} + \frac{16\mathcal{K}^2 k_\parallel}{s} \times \Big[\sinh(k_\parallel d)\sinh(sd) \times \Big(\frac{\mathcal{K}^2 k_\parallel}{\delta s}\sinh(k_\parallel d)\sinh(sd) + \delta^{1/2}(1+\delta)\Big)\Big] \quad (16)$$
$$+ \frac{4\cosh(2k_\parallel d)\cosh(2sd)}{\delta} - \frac{8\mathcal{K}^2 k_\parallel \sinh(2sd)\sinh(2k_\parallel d)}{\delta s},$$

$$R(k_{\|}, s) = -2\frac{\cosh(2sd)}{\delta} - \frac{2\cosh(2k_{\|}d)}{\delta^2} + \frac{8\mathcal{K}^2 k_{\|}\sinh(sd)\sinh(k_{\|}d)}{\delta^{3/2}}.$$

The roots of Eq. (15), along with expressions (16), relate spectral EIW parameters $k_{\|}$ and s to the Bloch wavenumber. Holding to the interpretation of the dispersion law as the functional coupling $s = s(k_{\|})$ accepted in the theory of electroacoustic waves [14, 15], it would be reasonable to consider Bloch wavenumber χ as an additional parameter. The occurrence of this parameter owes to the conditions of translational symmetry of the solution describing EIWs over the entire set of allowed values $-\pi/d < \chi < \pi/d$. Thus, at the designated χ value, Eq. (15), along with expressions (16), establishes the dispersion properties of a partial EIW.

Features of a spectrum of the partial EIWs confined by a moving lattice are determined by the velocity of DW motion. In Eq. (15), the parametric dependence on V_D is implicit and implemented through wave vector component $k_\perp$ from (7). In view of the inconvenience of the obtained expressions and, above all, transcendence of Eq. (15), the dependences of the EIW dispersion spectra on the velocity of DW motion can be demonstrated only numerically. However, multiplying Eq. (15) by δ and dividing it by λ^2 in accordance to Eq. (16), we obtain the simpler form:

$$\begin{aligned} &a^2 + a[4\cos(k_\perp d) + 4\cos(2\chi d - k_\perp d) \\ &- 8\cosh(sd)\cosh(k_{\|}d)] \\ &+ \cos(4\chi d - 2k_\perp d) - 4\cos(2\chi d)\cosh(2sd) \\ &- 4\cos(2\chi d - 2k_\perp d)\cosh(2k_{\|}d) \\ &+ 2\cos(2k_\perp d) + 4\cosh(2sd)\cosh(2k_{\|}d) = 0, \end{aligned} \tag{17}$$

where $a = 4k_{\|}\mathcal{K}^2\sinh(sd)\sinh(k_{\|}d)/s$. Then, solving Eq. (17) with respect to a, one can obtain a dispersion relation in the form more convenient for an analysis and numerical calculations. In particular, if the Bloch wavenumber $\chi = \pi/(2d)$ is taken (in the periodic structure only the χ values lying within the first allowed band $-\pi/d < \chi < \pi/d$ correspond to the physically nonequivalent states), from Eq. (17) we obtain

$$s = \mathcal{K}^2 k_{\|}\left\{\tanh(k_{\|}d)\tanh(sd) \times \frac{[1 \pm \sin^2(k_\perp d)/\cosh(sd)]}{[1 - \sin^2(k_\perp d)]/\cosh^2(sd)}\right\}. \tag{18}$$

Expression (18) shows, in particular, that, while for a static lattice ($k_\perp = 0$), only one mode exists, in the case of a moving lattice ($k_\perp \neq 0$) the spectrum splits into two different modes corresponding to the alternating "+" and "−" signs. This splitting can be considered as the removal of a degeneracy of the interaction of acoustoelecric oscillations of DWs of a unit cell due to their transverse motion taking place for $\chi = \pi/(2d)$.

To turn to the static lattice at $\chi \neq \pi/(2d)$, one should take $k_\perp = 0$ in Eq. (17). After solving the quadratic equations with respect to a and making some transformations, we obtain the explicit representation of the EIW dispersion equation:

$$s = -k_{\|}\mathcal{K}^2 \times \left\{\frac{\sinh(k_{\|}d)\sinh(sd)}{[\cos(\chi d) \mp \cosh(k_{\|}d)][\cos(\chi d) \pm \cosh(sd)]}\right\}. \tag{19}$$

According to Eq. (19), the partial EIW spectrum in a ferroelectric with the static lattice initially includes two modes for a fixed value of the Bloch wavenumber different from $\pi/(2d)$. This result could be expected from the results reported in [16] for a stripe domain. Note, however, that at finite values of the lattice period, no strict correspondence between the spectra of modes (17) and (19) in the limit $V_D \to 0$ and the spectra from [16] is observed due to the conditions of translational symmetry of solution (11). On the other hand, the transition to the results of [16] under the condition $d \to \infty$ is excluded from Eqs. (17) and (19) at all, as it requires additional consideration of another boundary problem for the structure of alternating domains with different thicknesses. The only available analytical confirmation of correspondence remains the fact that in the limit $d \to \infty$, when the transition to an individual DW occurs, from Eqs. (17) and (19) we immediately have the expected result $s = k_{\|}\mathcal{K}^2$ [14].

DISPERSION SPECTRA OF MODES OF NONCOLLINEAR ELECTROACOUSTIC INTERFACIAL WAVES OF A DYNAMIC SUPERLATTICE OF A FERROELECTRIC CRYSTAL

It is reasonable to start the study of spectra of the partial EIWs of a dynamic superlattice with the case $\chi = \pi/d$, when the electroacoustic oscillations of DWs distant from one another by a lattice period are inphase. One can see from Eq. (17) that the results for this case are identical to those for $\chi = 0$ and $\chi = -\pi/d$. Another feature is that the transition to the case $\chi = -\pi/d$ can be implemented by inversion of the velocity of DW motion: $V_D \to -V_D$. Physically, it means that the dispersion spectra of EIW modes under the condition of synchronism of the electroacoustic oscillations of DWs are independent of the chosen direction of DW motion; i.e., the lattice reveals reciprocity of the transverse distribution of fields and EIW propagation.

The values of the Bloch wavenumber $\chi = \pm\pi/d$ determine the boundaries of the first allowed band. A schematic of the dispersion EIW spectra for the case $\chi = \pm\pi/d$ is presented in Fig. 2. The dispersion branches showed by dashed lines correspond to the static lattice; the presence of DW motion is reflected by bold solid lines. Thin straight lines represent the linear EIW spectra on a single DW, static ($\beta = 0$) [14] or moving ($\beta \neq 0$) [9]. A dashed straight line shows the linear spectrum of volume SH waves propagating in a single-domain crystal.

The overall picture of the spectrum of modes of the partial EIWs at $\chi = \pm\pi/d$ outwardly resembles the picture of the spectrum of modes of a stripe domain: there are only two modes, and the high-frequency one has a lower cutoff frequency (black point in the figure); the low-frequency mode is present over the entire frequency range. The change in the high-frequency asymptotes of the spectra due to DW motion is the same as that for a single DW [9] or a stripe domain [10]. However, there are principle distinctions between the spectra of stripe domain modes and the spectra in Fig. 2.

First, as the velocity of DW motion increases, the cutoff point of the high-frequency mode in Fig. 2 shifts, as the arrow indicates, along the spectrum of volume waves towards lower frequencies, i.e., in the opposite direction as compared to the case of a moving stripe domain. A consequence of this behavior is the limiting situation at $\beta \to 1$, when the high- and low-frequency branches merge with the linear spectrum of volume waves (the Marfeld–Turnois asymptote at $\beta \to 1$ approaches the volume wave spectrum) and the region of EIW existence degenerates into the spectral line. This degeneracy of the EIW into an ordinary shear wave propagating in the direction of DW motion corresponds to "tripping" the shear waves and the moving DWs due to the absence of piezoelectric polarization charges [9]. This result is quite predictable, since the number of DWs moving at the velocity of sound is of no importance, as in the case of the superlattice.

Second, the low-frequency branch exhibits a qualitatively different character of dispersion. Indeed, at low frequencies the dispersion branch for a stripe domain occurs below the spectrum of volume waves but above the Marfeld–Turnois mode asymptote. In other words, the phase velocity of the wave lies between the values of the EIW velocity on a single DW and the velocity of a volume SH wave. Meanwhile, in the lattice spectrum the dispersion branch of the low-frequency mode lies completely below the Marfeld–Turnois asymptote. This type of EIWs for a static lattice was mentioned previously in [2], where the resemblance of their dispersion to the dispersion of flexural modes in thin planes was pointed out.

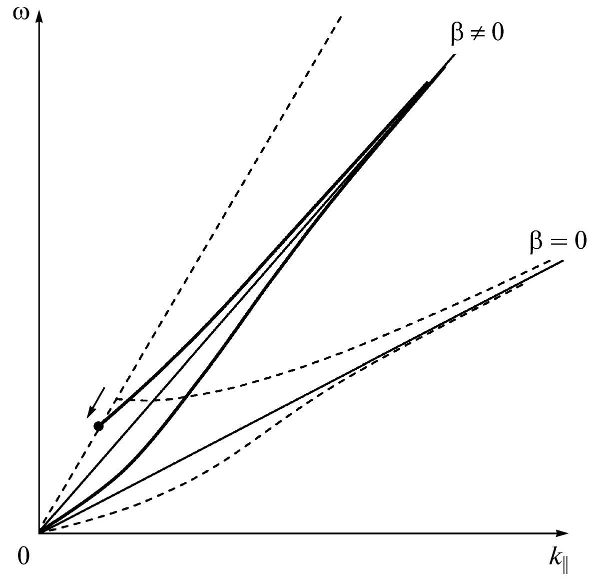

Fig. 2. General view of the mode spectrum of the partial EIW for the lattice with the Bloch wavenumbers $\chi = \pm\pi/d$.

Third, the mode spectra of a stripe domain correspond to different types of the transverse distribution of shear displacements, including symmetric for the low-frequency branch (symmetric mode) and antisymmetric for the high-frequency branch (antisymmetric mode). In the case of a lattice, the calculations using formulas (13) with the dispersion EIW indices obtained from Eq. (15) show that the distributions of the shear displacements across the unit cell are qualitatively similar for both modes and correspond mainly to the symmetric type. Obviously, the DW motion causes the symmetry break. Figure 1 demonstrates such a quasi-symmetric profile shown, for better illustration, in a strongly emphasized form.

When there are only two modes at $\chi = \pm\pi/d$, the degenerated case of EIW propagation takes place, i.e., there is no splitting of modes upon induced DW motion. Possibility of this splitting in other cases was analytically demonstrated in the previous section by the example of the lattice with $\chi = \pi/(2d)$. The shape of the partial EIW mode spectrum for the lattice with this Bloch wavenumber calculated from Eq. (19) is presented in Fig. 3.

As was mentioned above, in the static case Eq. (19) has only one root. The corresponding mode is shown in Fig. 3 by a dashed curve and is purely antisymmetric, judging by the character of the transverse distribution of the shear displacements. This naturally follows from the fact that, at $\chi = \pi/(2d)$, the electroacoustic oscillations on DWs distant by a lattice period are antiphase. The absence of a symmetric mode is

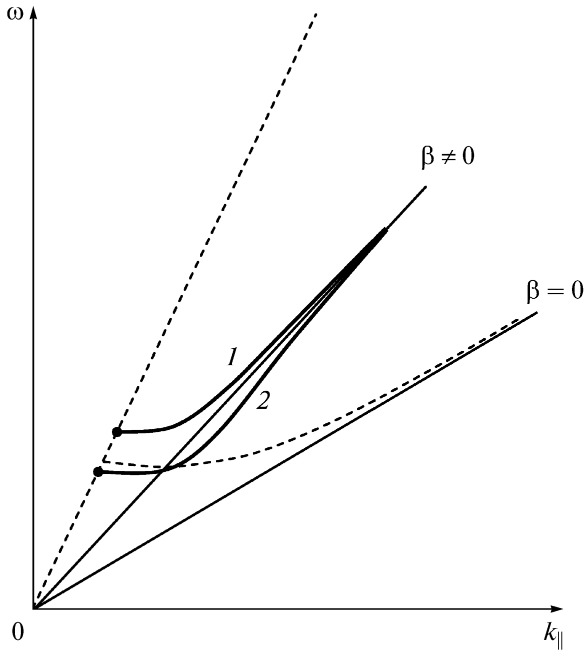

Fig. 3. Spectrum of the partial EIW modes for the Bloch wavenumber $\chi = \pi/(2d)$: *1* and *2* are the branches split by DW motion.

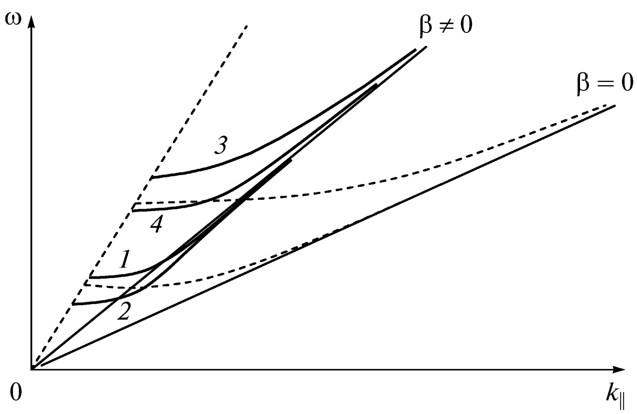

Fig. 4. Spectrum of the partial EIW modes for the arbitrary Bloch wavenumber $\chi \neq \pi/(2d)$ and $\chi \neq \pi/d$: *1*, *2* and *3*, *4* are the pair branches split by DW motion.

explained by piezoelectric inconsistency of the symmetrically distributed shear displacements and the resulting stresses in an internal DW of the unit cell due to the alteration of a sign of the piezoelectric modulus [11].

Motion of DWs splits the dashed branch of the antisymmetric mode into two branches *1* and *2* shown in Fig. 3 by bold solid lines. In the high-frequency limit, the spectra of the split modes are asymptotically drawn to the changed (turned toward the dashed line of the volume wave spectrum) Marfeld–Turnois asymptote. With the change in the direction of DW motion (alteration of signs in Eq. (19) from $\pm$ to $\mp$), branches *1* and *2* in Fig. 3 change places. Clearly, this does not break the overall picture of the spectrum. In this sense, the conclusion on reciprocity of the transverse field distribution and EIW propagation revealed by the lattice of moving DWs retains its validity in the considered variant.

One can see from Eq. (19) that the effect of the inversion of the velocity of DW motion on the spectrum of modes of the partial EIW is equivalent to that of the transformation $\chi \to -\chi$. Thus, the picture of the spectra in Fig. 3 includes the case $\chi = -\pi/(2d)$. This conclusion is general and can be extended to any case $\chi \neq \pi/(2d)$. The most typical situation of the transformation of the partial EIW mode spectrum by the lattice of moving DWs is that for $\chi \neq \pi/(2d)$ in the static case there are already two modes (dashed curves in Fig. 4) instead of the only mode split into pairs by motion, as in Fig. 3. The transformations $V_D \to -V_D$ or $\chi \to -\chi$ yield intrapair rearrangement of the split modes $1 \leftrightarrow 2$ and $3 \leftrightarrow 4$ (solid curves), which validates general independence of spectra representation on the direction of DW motion.

Concerning the structure of the transverse distribution of the shear displacements for the split branches, one should pay attention on its noticeable closeness to the transverse distribution for a generative mode of a partial EIW of the static lattice. In the general case, the generative mode reveals mixed signs of symmetry-antisymmetry occurring depending on closeness of value $2\chi d$ to values 2π or π. If the situation corresponds to that illustrated in Fig. 3, the transverse distributions of the displacements for both split modes slightly differ from a strictly symmetric form at high velocities of DW motion ($\beta > 0.7$) and from one another.

Figure 5 depicts the dependence of the frequency of an electroacoustic wave on the Bloch wavenumber for some fixed wavenumber. The dispersion curves shown by dashed lines correspond to the static lattice; in the presence of DW motion, they are shown by solid lines. The low-frequency group of curves *1* corresponds just to the EIWs. The high-frequency group of curves *2* separated from group *1* by the band gap corresponds to the first allowed band for the electroacoustic waves of volume propagation [17] (presented for comparison). There is the only allowed band for the EIWs; it lies below all the allowed bands of the volume propagation spectra. In both cases, DW motion splits the spectrum of any electroacoustic wave of the static lattice into pairs (spectral doublet). In the general case $\chi \neq \pi/(2d)$ and $\chi \neq \pi/d$ for the EIWs at the chosen Bloch number and wave vector, we have four branches, as not one but two modes corresponding to different symmetries of the distribution of the shear displacements across the cell undergo pair splitting.[1] One can

[1] For the regions not close to $\chi = \pi/(2d)$ and $\chi = \pi/d$, two solid curves lying above in frequency correspond to the quasi-symmetric distribution.

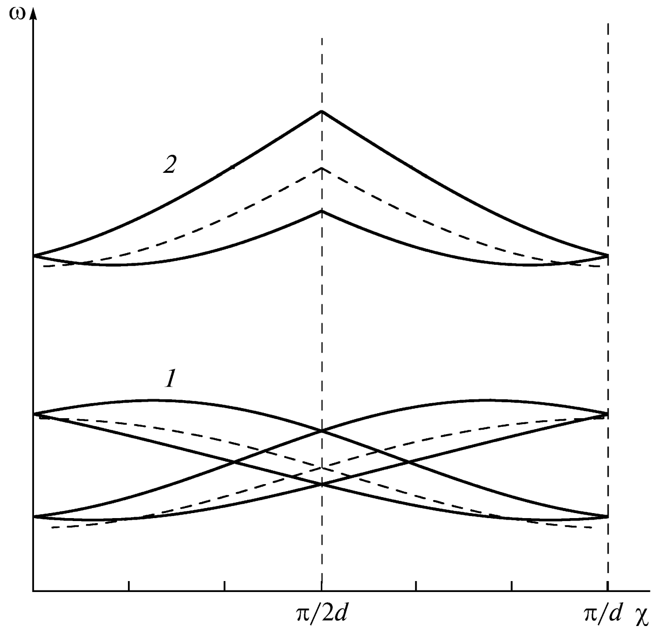

Fig. 5. Dependence of the frequency of an electroacoustic wave on the Bloch wavenumber at a fixed $k_{||}$ value.

see from Fig. 5 that, although the values of the Bloch number $\chi = \pm\pi/d$ determine the boundaries of the first allowed band, they do not determine the frequency region of location of the dispersion branches of all the rest of the partial EIWs. There exist intermediate Bloch wavenumbers for which the frequency at a fixed wavenumber lies above or below as compared to the case of the Bloch wavenumber $\chi = \pm\pi/d$. This feature is more pronounced for the dynamic lattice.

CONCLUSIONS

The dispersion properties of the noncollinear EIWs of the dynamic superlattice of the equidistant uniformly moving 180° DWs of ferroelectric crystals have been described. It has been shown that due to DW motion the partial Bloch spectra of modes of the interfacial electroacoustic waves that are not related to the boundaries of the first allowed band undergo pair splitting into high- and low-frequency branches, changing places during the inversion of the DW velocity. The partial modes setting the limits of the allowed band for the EIWs are not subjected to the mentioned splitting, invariant to the inversion of the velocity of DW motion, and transformed by DW motion in the high-frequency asymptote similarly to the waves on individual domain walls.

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