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Dispersion Properties of Two-Dimensional Phonon Crystals with a Hexagonal Structure

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Abstract—Acoustic and optical properties of two-dimensional phonon crystals with a hexagonal symmetry are described. Differential-difference equations describing plane oscillations of a two-dimensional lattice of material structures are derived in a harmonic approximation, and dispersion dependences of acoustic and optical phonons are calculated. Branches of optical oscillations and intervals of forbidden frequencies are shown to appear when a lattice consists of two types of atoms.

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INTRODUCTION

At present, propagations and interactions of acoustic waves in lattices and periodic structures are actively being investigated. This concerns, in particular, the study of phonons in nanostructures [1–3]. The keen interest in such types of materials is attributed to their unique properties that allows them to be used in many fields, in particular, in nanoelectronics.

The object of investigation was a metamaterial having a complex structure with two particles with different masses in a unit cell. In the case in which the masses of the particles are identical, the metamaterial structure coincides with that of a synthetic opal SiO_2, it being a crystalline structure from densely packed spheres with a diameter from 200 to 1000 nm [6, 7]. Opals were found to be characterized by anisotropy of thermal conductivity related to the passage of high-frequency acoustic phonons and the existence of "audio channels." No convincing theoretical explanation of these effects has been obtained as yet. In crystal acoustics, they usually work with continual models of anisotropic solids [8, 9], which are applicable only for waves with a length much larger than the lattice period. Such models cannot describe the dispersion properties of short acoustic waves when the periodicity of the lattice starts and, thus, they cannot be used to explain formation of audio channels. A description of the dispersion properties of shortwave perturbations necessitates having a mathematical apparatus developed in lattice dynamics and solid state physics [10, 11].

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A HEXAGONAL LATTICE WITH ONE PARTICLE IN A CELL

Consider oscillations of a hexagonal lattice (Fig. 1) with homogeneous particles with the mass M positioned in the lattice points. In the initial state, the centers of the particle mass are in the lattice points, the distance between them being a. Each particle interacts with six immediate neighbors the centers of mass of which are at the apexes of a right hexagon inscribed into a circumference with the radius a (the first coordination sphere). The lattice points are numbered with respect to the central points ($n = 1, ..., 6$) anticlockwise staring from the x axis. Each particle is characterized by two degrees of freedom: displacements of the centers of mass $u_n(t)$ and $w_n(t)$ along the x and y axes, respectively, where n is the point number. The poten-

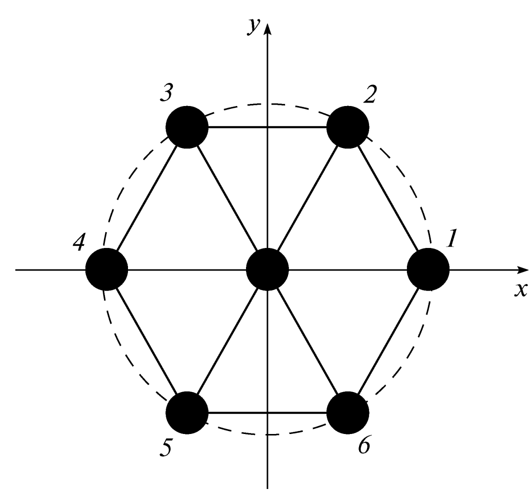


Fig. 1. A scheme of particle interaction in a hexagonal lattice.

tial energy in each lattice cell is a function of the momentary positions of its particles. It can be represented as a sum of energies of pairwise central interactions between the lattice particles that depend only on the distances between them [8]. In the case of structural modeling, an equivalent power scheme in the form of a system of rods or springs responsible for the transfer of forces and moments is usually introduced instead of a field description of the interaction [7, 10]. As all forces in the considered lattice are central, a spring model is used for simulation of the interaction. In the harmonic approximation, the potential energy per lattice cell is

$$U = \frac{k}{2}\sum_{n=1}^{6} D_n^2,$$

where D_n is the variation in the distances (spring lengths) between the particle positioned in the center of a configuration sphere and its neighbors ($n = 1, ..., 6$). The multiplier 1/2 in front of the sum sign takes into account that the potential energy of each spring is equally divided between the two particles connected with this spring. In the approximation of the smallness of particle displacements, the equations for D_n have the form

$$D_n = \cos\left((n-1)\frac{\pi}{3}\right)\Delta u_n + \sin\left((n-1)\frac{\pi}{3}\right)\Delta w_n,$$

where $\Delta u_n = u_n - u_c$, $\Delta w_n = w_n - w_c$, u_c, w_c is the displacement of the central point. As the lattice in the considered case is simple, the kinetic energy of the cell is equal to the kinetic energy of its central particle,

$$T = \frac{m}{2}(u_c'^2 + w_c'^2).$$

Based on the variational principle of the Hamiltonian, we find the equations that describe the dynamics of a two-dimensional hexagonal structure,

$$Mw''_{tt} = \frac{\sqrt{3}k}{4}\sum_{\substack{n=2\\ n\neq 4}}^{6}(\Delta u_n - \sqrt{3}\Delta w_n),$$
$$Mu''_{tt} = \frac{k}{4}\sum_{\substack{n=2\\ n\neq 4}}^{6}(\Delta u_n - \sqrt{3}\Delta w_n) + \frac{k}{4}\sum_{n=1,4}\Delta u_n. \quad (1)$$

Let us consider now some properties of elastic wave propagation in such a periodic system without imposing any limitations on the wavelengths of elastic oscillations. The wavelength can be both much larger than and comparable with the distance between the particles if a conventional continual approximation does not work.

DISPERSION RELATIONS

All particles in the considered structure are physically equivalent, and thus excitation of any particle is to be redistributed over the entire structure. In other words, each motion of an individual particle will stimulate corresponding motions of neighboring particles. It will yield running of a wave over the lattice, it being a typical collective motion. Let us turn to normal oscillations to consider collective motions in a crystal that is regarded as an ordered collective [8, 9]. Consider the solutions to Eq. (1), which represent plane monochromatic waves for which the displacement can be represented as follows:

$$u(\mathbf{N}, t) = A\exp[i(\omega(\mathbf{q})t - \mathbf{qN})],$$
$$w(\mathbf{N}, t) = B\exp[i(\omega(\mathbf{q})t - \mathbf{qN})], \quad (2)$$

where $\omega = \omega(\mathbf{q})$ is the wave frequency considered as a continuous function of the wave vector $\mathbf{q} = (q_1, q_2)$ specifying the direction of wave propagation in the Cartesian coordinate system (x, y) and its length $\lambda = 2\pi/q$, $q = |\mathbf{q}|$ is the wave vector modulus. The components of the wave vector are given in a polar coordinate system, $\mathbf{q} = (q\cos\varphi, q\sin\varphi)$, and φ is the angle between the wave vector direction and the x axis. The quantities A and B determine displacement amplitude in a wave along the coordinate axes. The vector $\mathbf{N}$ numerates the lattice points. Arbitrary collective motions can be represented as a superposition of monochromatic waves.

Substitution of the solutions to Eq. (2) into Eq. (1) yields a set of equation for determining displacement amplitudes which can be represented in a matrix form,

$$\begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = 0,$$

$$d_{11} = M\omega^2 - 3k + 2k\cos(q_1 a) + \sqrt{3}k\cos\left(\frac{q_1 a}{2}\right)\cos\left(\frac{q_2\sqrt{3}a}{2}\right),$$

$$d_{12} = d_{21} = \sqrt{3}k\sin\left(\frac{q_1 a}{2}\right)\sin\left(\frac{q_2\sqrt{3}a}{2}\right),$$

$$d_{22} = M\omega^2 - 3k + \sqrt{3}k\cos\left(\frac{q_1 a}{2}\right)\cos\left(\frac{q_2\sqrt{3}a}{2}\right).$$

The set of equations has a nontrivial solution if its determinant is $d(\omega, q_1, q_2) = 0$, which yields a dispersion correlation. The dispersion equation is a biquadratic with respect to frequency, and from the geometrical point of view describes a family of two surfaces periodic over q_1 and q_2 in a three-dimensional space (ω, q_1, q_2). A proper normal mode of oscillations corresponds to each of the surfaces. Two independent waves (normal modes) with mutually perpendicular displacements correspond to any wave vector direc-

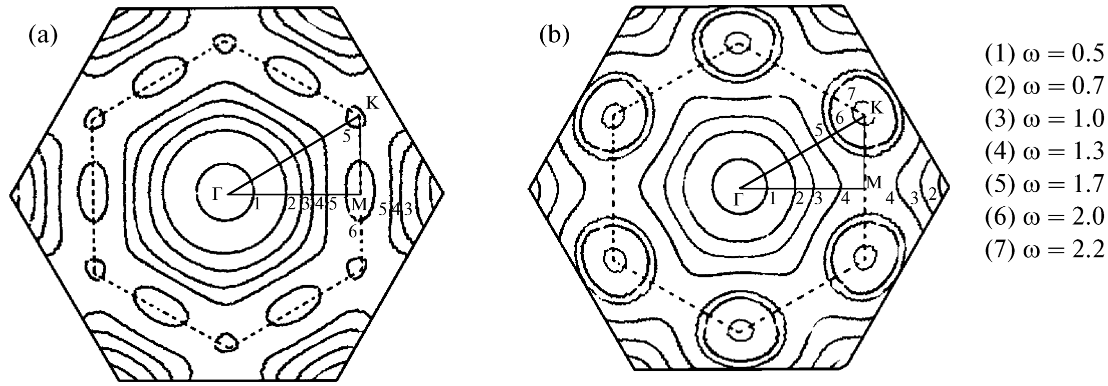


Fig. 2. The pattern of equal frequencies for longitudinal (a) and transverse (b) phonons.

tion. Based on the direction of displacement with respect to the wave vector, the oscillations can be divided into longitudinal (LA when the displacement is parallel to **q**) and transverse (TA when the displacement is perpendicular to **q**). Strictly speaking, displacements of particles during oscillations are either parallel or perpendicular to the vector **q** only during wave propagation in the symmetry directions. In the general case, the waves are not purely longitudinal or purely transverse [9, 12]. The velocity of propagation of longitudinal oscillations is, as a rule, higher than that of transverse ones and, thus, a mode with a higher phase velocity in solid acoustics is called longitudinal and that with a lower phase velocity is called transverse. When a displacement wave propagates at the angle φ to the x axis, the longitudinal A_L and the transverse A_T component of displacement are expressed through the amplitudes A and B,

$$A_L = A\cos\varphi + B\sin\varphi, \quad A_T = -A\sin\varphi + B\cos\varphi.$$

It follows from the equation for determining displacement amplitudes that displacements of A and B are related by the equation $A = -(d_{12}/d_{11}(\omega_k))B$. We can introduce the coefficients of propagation of longitudinal and transverse displacements in the acoustic branch of oscillations $\omega = \omega(\mathbf{q})$ ($k = 1, 2$) that satisfy the normalization condition $(k^L)^2 + (k^T)^2 = 1$,

$$k^L(\omega_k, \varphi) = \left.\frac{A_L}{\sqrt{A_L^2 + A_T^2}}\right|_{\omega_k} = \left.\frac{d_{11}\sin\varphi - d_{12}\cos\varphi}{\sqrt{d_{11}^2 + d_{12}^2}}\right|_{\omega_k},$$

$$k^T(\omega_k, \varphi) = \left.\frac{A_T}{\sqrt{A_L^2 + A_T^2}}\right|_{\omega_k} = \left.\frac{d_{11}\cos\varphi + d_{12}\sin\varphi}{\sqrt{d_{11}^2 + d_{12}^2}}\right|_{\omega_k}.$$

Using these relations it can be shown that the waves propagating along the symmetry directions are purely longitudinal and purely transverse.

ANALYSIS OF DISPERSION PROPERTIES OF ACOUSTIC PHONONS

By analogy with solid state physics, each normal oscillation of the lattice corresponds to a quasiparticle—namely, a phonon. A longitudinal phonon corresponds to a longitudinal mode and a transverse phonon corresponds to a transverse mode [9]. Lattice periodicity yields a periodic dependence of frequency on the wave number and the notion of Brillouin zones [8, 9]. Figures 2 and 3 exemplify the patterns of equal frequencies and dispersion curves along the symmetry directions of a crystalline structure. During graphical plotting it was assumed that the dimensionless parameters of the lattice are as follows: $a = a_p/a_0 = 1$, $m = m_p/m_0 = 1$, and $k = k_p/k_0 = 1$, where the quantities with the index "0" represent dimensional scales of a real lattice. The patterns of equal frequencies shown in Fig. 2 are plane cross sections of three-dimensional surfaces $\omega_k(q_1, q_2)$ = const separately for longitudinal (Fig. 2a) and transverse (Fig. 2b) phonons. A dashed line in the figure marks the first Brillouin zone which represents itself an equilateral hexagon rotated by the angle $\pi/6$ with respect to the cell of a straight lattice (Fig. 1). With an increase in frequency, the level lines deviate from the circular one and take the shape of a hexagon, which points to the appearance of acoustic anisotropy in the crystalline structure. The existence of closed curves with centers lying on the boundaries of the Brillouin zone points to nonmonotony of the dispersion dependence.

Figure 3 shows the dispersion curves calculated along the symmetry directions (Γ–K and Γ–M) and along the boundary of the Brillouin zone (K–M) (see Fig. 2). The angle φ points to the direction of a plane wave propagation with respect to the x axis in a straight lattice (Fig. 1). It is apparent that the frequency of longitudinal phonons in the Γ-K direction has a maximum at the point $q = 2(\pi - \arctan(3\sqrt{7}))/a$ and then decreases until the boundary of the Brillouin zone. It results in the fact that there is a section on the disper-

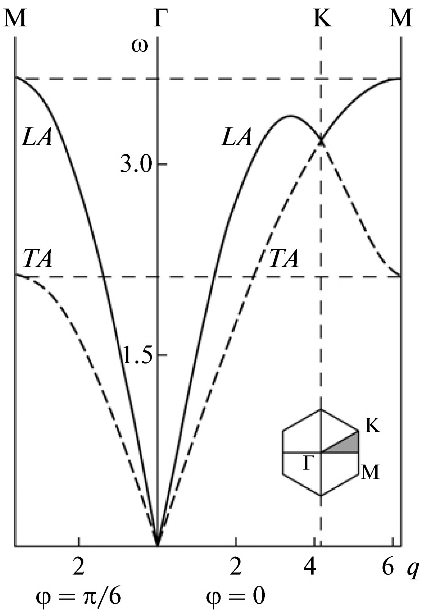


Fig. 3. Dispersion curves for longitudinal and transverse phonons in the lattice.

sion curve where the group velocity is negative, $V_{gr} = d\omega/dq < 0$. This is the so-called region of a backward wave. Backward waves in periodic structures are associated with the phenomenon of negative refraction in case the surfaces of equal frequencies are convex [6]. As a rule, the region of a negative group velocity is observed for the dispersion branches of "optical" phonons in lattices with a complex structure when there is more than one particle in the periodicity cell. Here, an analogous situation takes place for optical phonons in a simple lattice that has not been observed earlier.

Anisotropy of the phonon spectrum is determined by the anisotropy of phase velocities $V_{\text{ph}} = \omega/|\mathbf{q}|$, which depend on the angular variable φ and the wave vector **q**. Figure 4 shows angular dependences of phase velocities of transverse and longitudinal waves at different wavelengths.

It is apparent from the plots that the behavior of phase velocities and, as a result, phonon spectra substantially differs from similar characteristics for anisotropic media. In the long-wave region, phase velocities do not depend on the angle and, thus, the lattice behaves as an isotropic medium. This fact is well known in the mechanics of a solid deformed body. However, when the wavelength starts decreasing, the phase velocities become angle-dependent (Fig. 4b), i.e., the properties of anisotropy show up as related to shortwave phonons. At wavelengths comparable with the distance between the particles in a lattice, so-called "audio channels" are formed. The phonon propagation velocity in these channels is much higher than the velocities of phonon propagation in other directions (Fig. 4b). These results are experimentally verified based on investigations of thermal conductivity on crystalline structures [3].

A HEXAGONAL CRYSTALLINE LATTICE WITH TWO PARTICLES IN A CELL

Let us consider a crystalline metamaterial with two point particles in a unit cell (Fig. 5). The material has a hexagonal structure with the distance between the particles equal to a. The geometrical structure of the metamaterial can be represented in the form of two interpenetrating lattices in the points of which there are particles of two types with different masses M_r, $r = 1, 2$. The kinetic energy of a cell is equal to the kinetic energy of the particles in it,

$$T = \frac{M_1}{2}(\dot{u}_1^2 + \dot{w}_1^2) + \frac{M_2}{2}(\dot{u}_2^2 + \dot{w}_2^2).$$

To calculate the potential energy, we will take into account the interaction of central particles with numbers 1 and 2 with the neighboring particles lying on the nearest coordination sphere, i.e., the neighbors that are nearest to this point. For this, let us introduce three types of springs with rigidities k_1, k_2, and k_{12}. The springs with the rigidity k_1 connect only the particles with the mass M_1; with the rigidity k_2, only the particles with the mass M_2; and with the rigidity k_{12}, the particles with different masses M_1 and M_2. We will hold to one and the same principle of denomination as in case of a simple hexagonal structure with one mass.

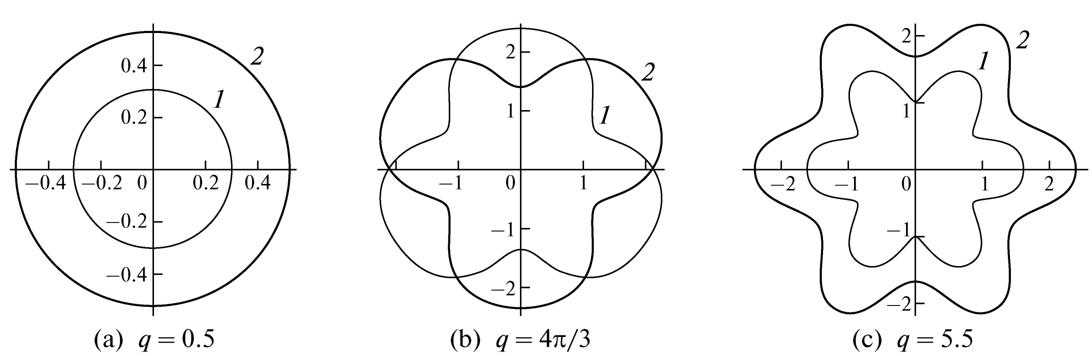


Fig. 4. Angular dependences of normal mode velocities at different wavelengths.

Let us number the particles lying on one coordination sphere anticlockwise with respect to the central particle starting from the x axis. The particles belonging simultaneously to two coordination spheres will have a double numeration.

The potential energy in one lattice cell (r is the number of the central point, n is the number of the particle on a corresponding coordination sphere, and $n = 3, 6$ correspond to the direction of a chain of identical atoms) is

$$U = \frac{k_1}{2}\sum_{n=3,6} D_{n1}^2 + \frac{k_2}{2}\sum_{n=3,6} D_{n2}^2 + \frac{k_{12}}{2}\sum_{r=1}^{2}\sum_{\substack{n=1\\ n\neq 3}}^{5} D_{nr}^2,$$

where $D_{nr} = \cos\left((n-1)\frac{\pi}{3}\right)\Delta u_{nr} + \sin\left((n-1)\frac{\pi}{3}\right)\Delta w_{nr}$.

The equations describing the dynamics of a two-dimensional hexagonal lattice have the form

$$\begin{aligned}
M_1 u1''_{tt} &= \frac{k_{12}}{4}\sum_{n=1,4}\Delta u_{n1} + \frac{k_1}{4}\sum_{n=3,6}(\Delta u_{n1} - \sqrt{3}\Delta w_{n1}) \\
&\quad + \frac{k_{12}}{4}\sum_{n=2,5}(\Delta u_{n1} - \sqrt{3}\Delta w_{n1}), \\
M_1 w1''_{tt} &= \frac{\sqrt{3}k_1}{4}\sum_{n=3,6}(\Delta u_{n1} - \sqrt{3}\Delta w_{n1}) \\
&\quad + \frac{\sqrt{3}k_{12}}{4}\sum_{n=2,5}(\Delta u_{n1} - \sqrt{3}\Delta w_{n1}), \\
M_2 u2''_{tt} &= \frac{k_{12}}{4}\sum_{n=1,4}\Delta u_{n2} + \frac{k_2}{4}\sum_{n=3,6}(\Delta u_{n2} - \sqrt{3}\Delta w_{n2}) \\
&\quad + \frac{k_{12}}{4}\sum_{n=2,5}(\Delta u_{n2} - \sqrt{3}\Delta w_{n2}), \\
M_2 w2''_{tt} &= \frac{\sqrt{3}k_2}{4}\sum_{n=3,6}(\Delta u_{n2} - \sqrt{3}\Delta w_{n2}) \\
&\quad + \frac{\sqrt{3}k_{12}}{4}\sum_{n=2,5}(\Delta u_{n2} - \sqrt{3}\Delta w_{n2}).
\end{aligned} \quad (3)$$

Here, $n = 1, 4$ correspond to the number of the particle on the coordination sphere in the direction along the x axis, $n = 2, 5$ correspond to the direction at the angle of $\pi/3$ with respect to the x axis. Let us seek the solution to set of equations (3) in the form of plane monochromatic waves ($\alpha = 1, 2$)

$$u_\alpha(\mathbf{N}, t) = A_\alpha \exp[i(\omega(\mathbf{q})t - \mathbf{qN})],$$
$$w_\alpha(\mathbf{N}, t) = B_\alpha \exp[i(\omega(\mathbf{q})t - \mathbf{qN})].$$

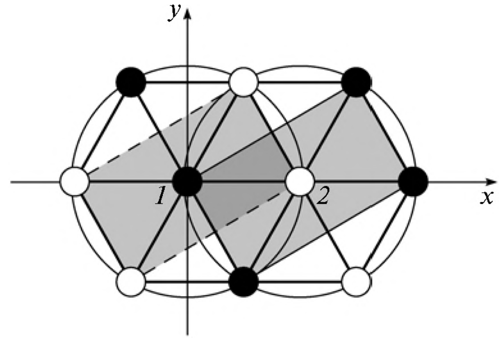


Fig. 5. A scheme of particle interaction in a hexagonal lattice with atoms of two types.

Then set of equations (3) is transformed to a matrix form for determining displacement amplitudes:

$$\begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \\ B_1 \\ B_2 \end{pmatrix} = 0, \quad (4)$$

where the matrix elements have the form

$$d_{11} = M_1\omega^2 + \frac{k_1(\cos(\theta_1) - 1)}{2} - \frac{5k_{12}}{2},$$

$$d_{12} = d_{21} = \frac{\sqrt{3}(k_1 - k_{12})}{2} - \frac{\sqrt{3}k_1\cos(\theta_1)}{2},$$

$$d_{13} = d_{31} = \frac{k_{12}\cos(\theta_2)}{2} + 2k_{12}\cos(q_1 a),$$

$$d_{14} = d_{23} = d_{32} = d_{41} = \frac{\sqrt{3}k_{12}\cos(\theta_2)}{2},$$

$$d_{22} = M_1\omega^2 + \frac{3k_1(\cos(\theta_1) - 1)}{2} - \frac{3k_{12}}{2},$$

$$d_{24} = d_{42} = \sqrt{3}d_{14},$$

$$d_{33} = M_2\omega^2 + \frac{k_2(\cos(\theta_1) - 1)}{2} - \frac{5k_{12}}{2},$$

$$d_{34} = d_{43} = \frac{\sqrt{3}(k_2 - k_{12})}{2} - \frac{\sqrt{3}k_2\cos(\theta_1)}{2},$$

$$d_{44} = M_2\omega^2 + \frac{3k_2(\cos(\theta_1) - 1)}{2} - \frac{3k_{12}}{2},$$

$$\theta_1 = \frac{q_1 - \sqrt{3}q_2}{2}, \quad \theta_2 = \frac{q_1 + \sqrt{3}q_2}{2}.$$

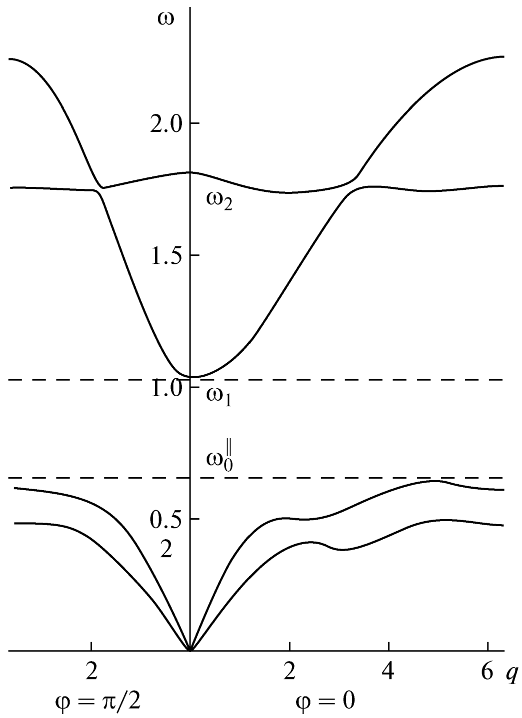


Fig. 6. Directions corresponding to alternation of masses.

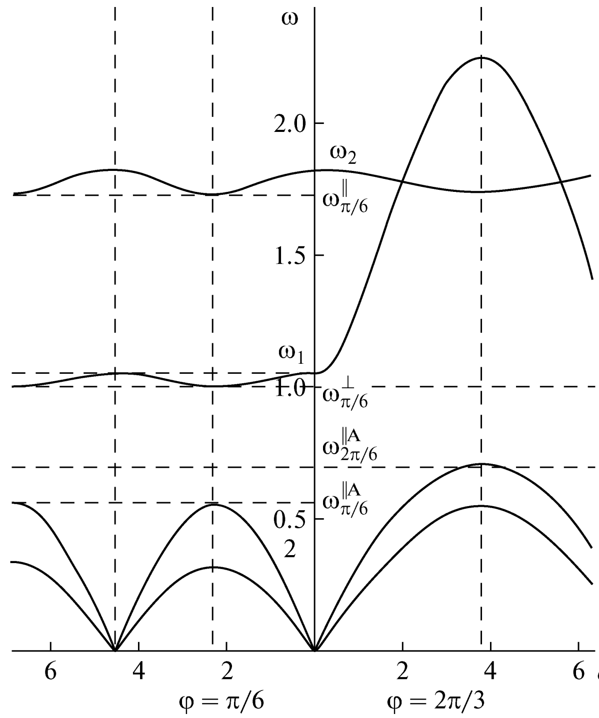


Fig. 7. Directions corresponding to the chains of identical atoms.

The condition of solvability of Eq. (4) yields a dispersion equation that is not given here due to its inconvenience.

Let us consider a derived set of equations (4) for the case of equal particle masses, i.e. $M_1 = M_2 = M$. Then, the rigidities of the springs should also be $k_1 = k_2 = k_{12} = k$. The matrix elements are transformed to the form

$$d_{11} = d_{33} = M\omega^2 + \frac{k\cos(\theta_1)}{2} - 3k,$$

$$d_{12} = d_{21} = d_{34} = d_{43} = -\frac{\sqrt{3}k\cos(\theta_1)}{2},$$

$$d_{13} = d_{31} = \frac{k\cos(\theta_2)}{2} + 2k\cos(q_1 a),$$

$$d_{14} = d_{23} = d_{32} = d_{41} = \frac{\sqrt{3}k\cos(\theta_2)}{2},$$

$$d_{24} = d_{42} = \sqrt{3}d_{14},$$

$$d_{22} = d_{44} = M\omega^2 + \frac{3k\cos(\theta_1)}{2} - 3k.$$

In this case, set of equations (1) will have a nontrivial solution if only the condition $A_1 = A_2$ and $B_1 = B_2$ is met. Then it degenerates into set of equations (2) for a hexagonal lattice containing particles with one and the same mass.

ANALYSIS OF DISPERSION PROPERTIES OF ACOUSTIC AND OPTICA PHONONS

In the chosen metamaterial model let us consider the effect of the particle mass ratio M_2/M_1 on dispersion characteristics of the material. That is why we will consider rigidities of the springs used to simulate the interaction of particles equal, i.e., $k_1 = k_2 = k_{12} = k$.

Several symmetry directions can be distinguished in the structure, namely the directions corresponding to alternation of masses ($\varphi = 0$, $\varphi = \pi/3$) and perpendicular to them ($\varphi = \pi/2$, $\varphi = 5\pi/6$), the directions corresponding to the chains of identical atoms ($\varphi = 2\pi/3$) and perpendicular to them ($\varphi = \pi/6$). The angle φ shows the direction of a plane wave propagation with respect to the x axis. Figures 6 and 7 plot dispersion relations for these directions. They were constructed for the given lattice parameters: $a = 1$, $M_1 = 1$, $k = 1$, $M_2/M_1 = 10$. It is seen from the plots that there are both acoustic and optical phonons in the structure, At $q = 0$ optical phonons can oscillate at the frequencies equal to

$$\omega_1 = \sqrt{\frac{(M_1 + M_2)k}{M_1 M_2}} \text{ and } \omega_2 = \sqrt{3}\sqrt{\frac{(M_1 + M_2)k}{M_1 M_2}}.$$

Let us analyze the dispersion relation at different values of M_2/M_1 (see Figs. 8 and 9). It is apparent from

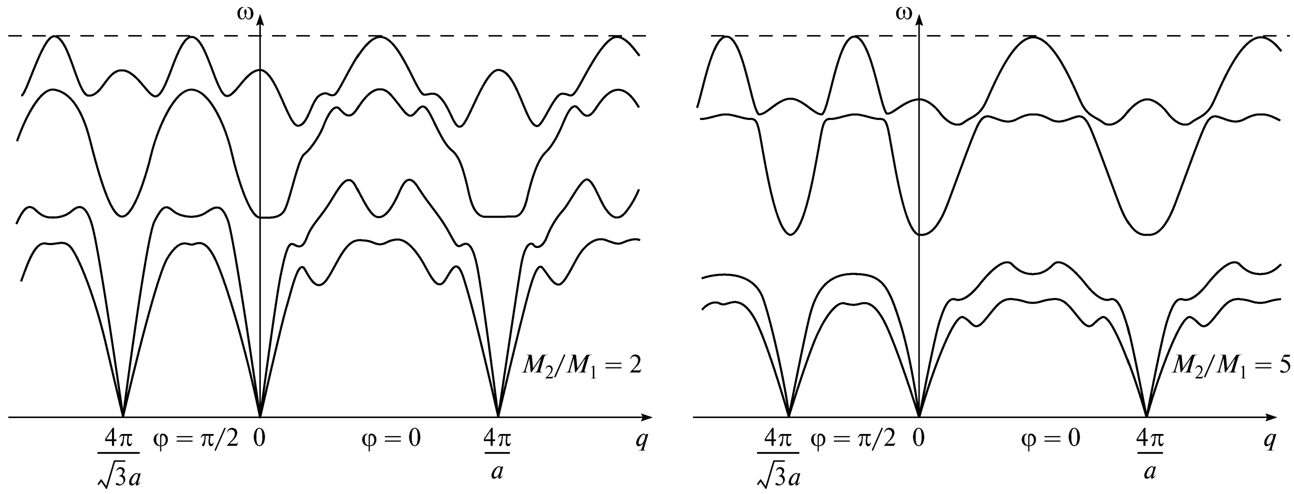


Fig. 8. Directions corresponding to alternation of masses.

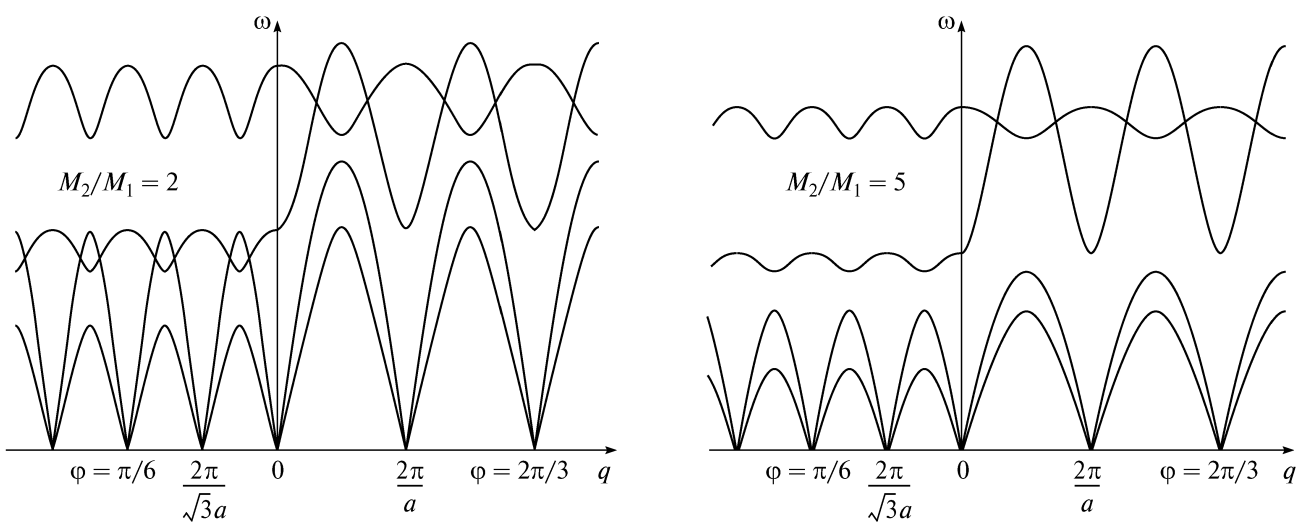


Fig. 9. Directions corresponding to the chains of identical atoms.

Figs. 8 and 9 that, with an increase in the particle mass ratio M_2/M_1, there appears a band of forbidden frequencies between the intervals of allowed frequencies the width of which can be calculated as minimal in all directions. Thus, the band width is

$$\Delta = \omega^{\perp}_{\pi/6} - \omega^{\|A}_{2\pi/3} = \sqrt{\frac{k}{M_1}} - \sqrt{\frac{5k}{M_2}},$$

where $\omega^{\perp}_{\pi/6}$ is the minimum of the frequency of transverse optical oscillations in the direction $\pi/6$ and $\omega^{\|A}_{2\pi/3}$ is the maximum of longitudinal acoustic oscillations in the direction $2\pi/3$. In the direction $\varphi = \pi/6$, there also appears an interval of forbidden frequencies,

$$\Delta_{\pi/6} = \omega^{\|}_{\pi/6} - \max(\omega_1, \omega^{\|A}_{\pi/6})$$
$$= \sqrt{\frac{3k}{M_1}} - \max\left(\sqrt{\frac{(M_1 + M_2)k}{M_1 M_2}}; \sqrt{\frac{3k}{M_2}}\right).$$

Figures 10 and 11 show patterns of equal frequencies along the same directions of a crystalline structure symmetry. These patterns of equal frequencies are plane cross sections of three-dimensional surfaces $\omega = \omega(q)$ separately for longitudinal and transverse acoustic phonons (Fig. 10) and separately for longitudinal and transverse optical phonons (Fig. 11). The Brillouin zones can be distinguished in the figures. The zones are rectangles rotated by the angle $\pi/2$ with

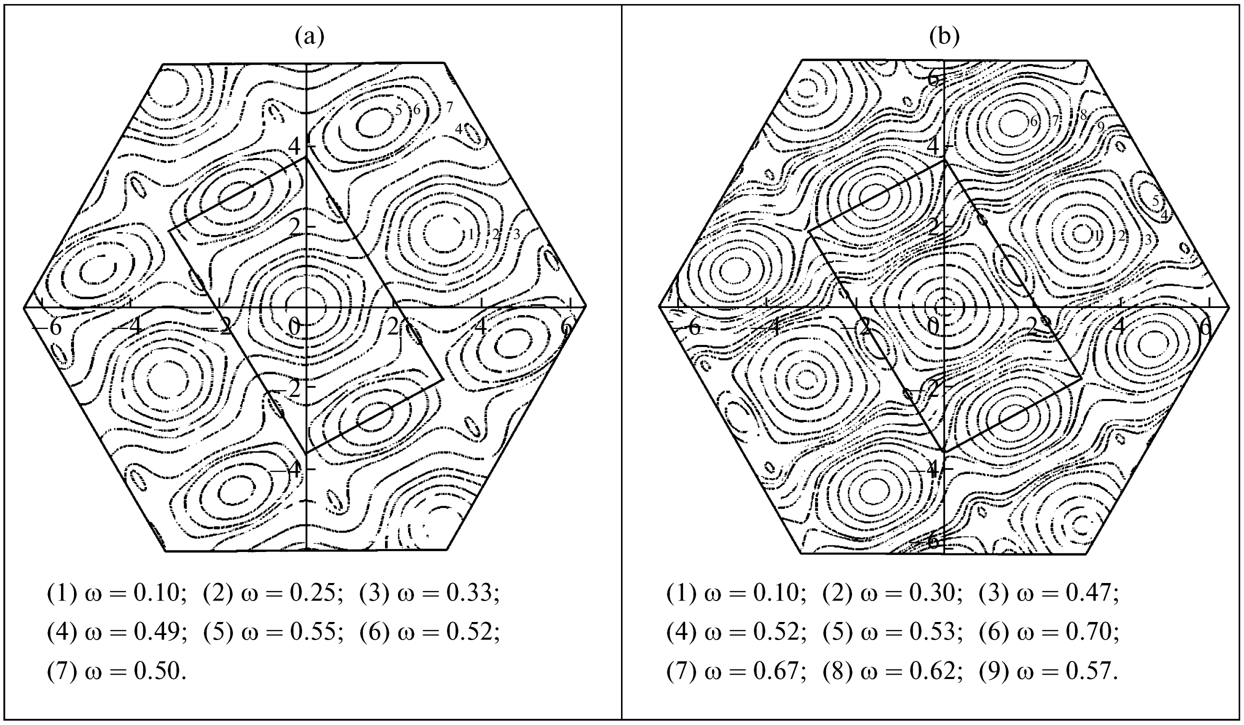


Fig. 10. The pattern of equal frequencies for transverse (a) and longitudinal (b) acoustic phonons.

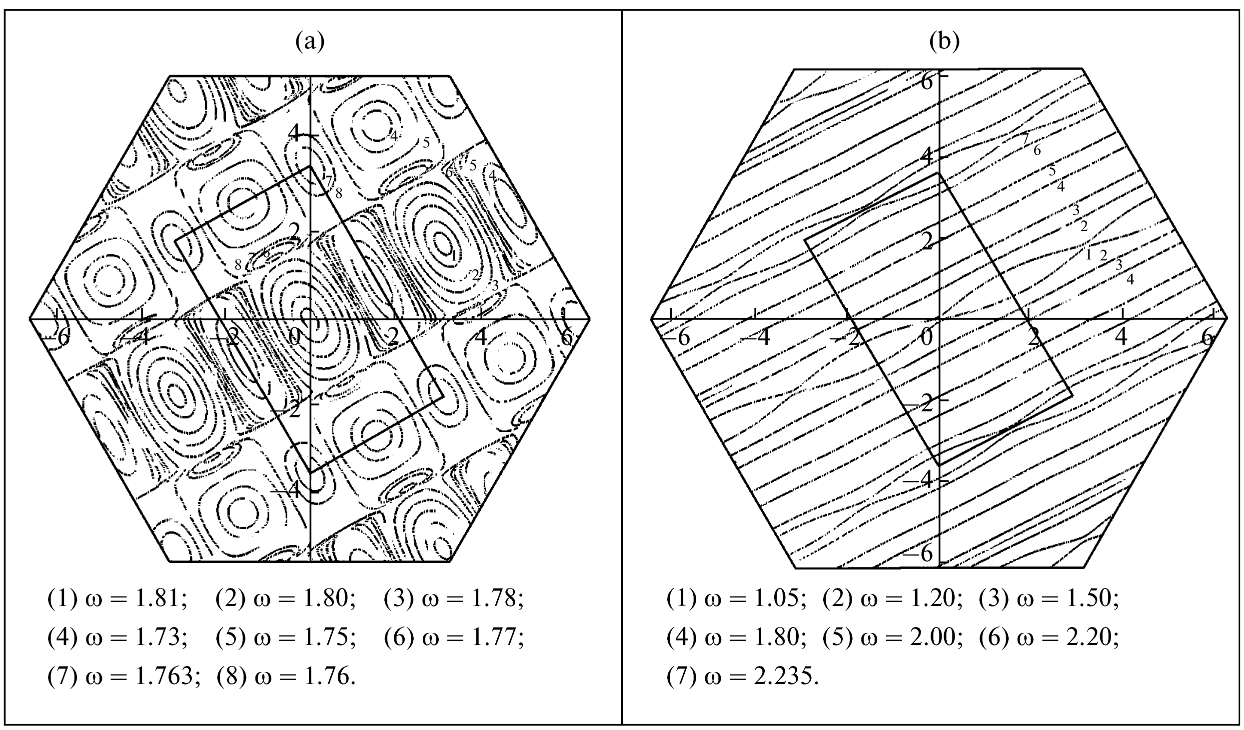


Fig. 11. The pattern of equal frequencies for transverse (a) and longitudinal (b) acoustic phonons.

respect to the cell of a straight lattice (Fig. 5). Analyzing the patterns of different frequencies for a single-mass (Fig. 2) and a double-mass (Figs. 10, 11), lattices we can conclude that the geometry of the Brillouin zone repeats that of the lattices which simulate the structure. The surfaces $\omega = \omega(q)$ are

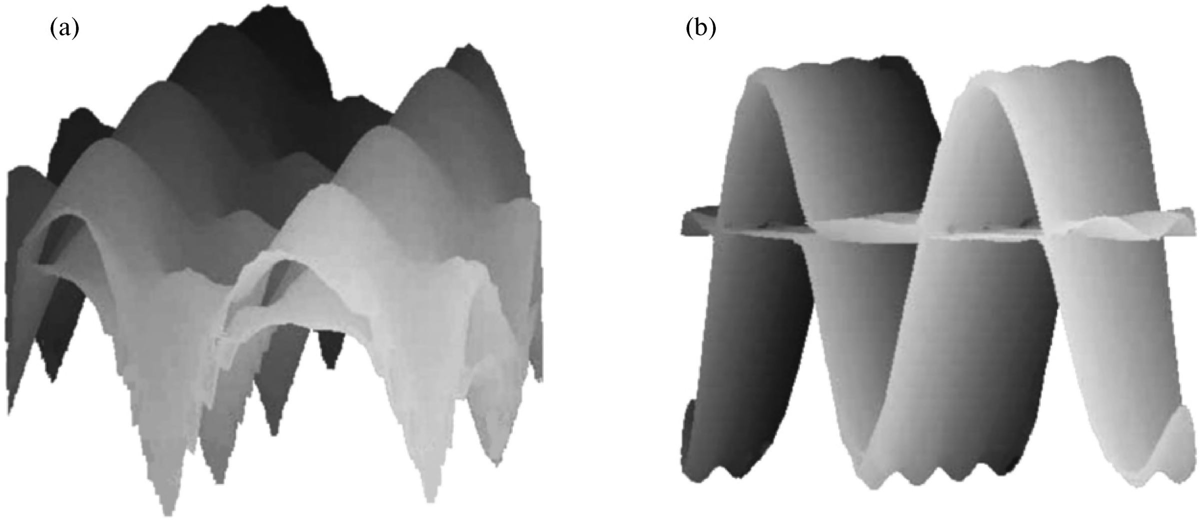
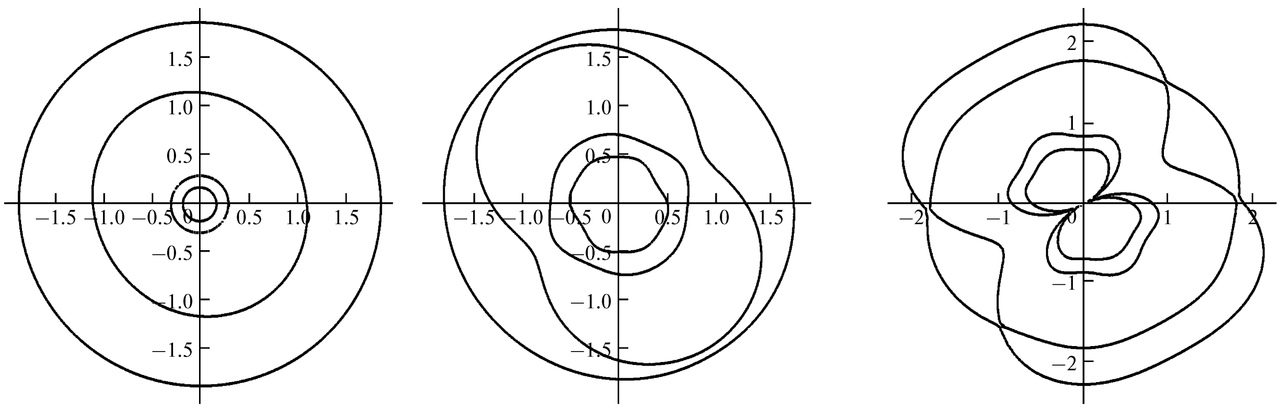


Fig. 12. The surfaces $\omega(k)$ for acoustic (a) and optical (b) phonons.



(a) $|q| = 0.5$ (b) $|q| = \pi/2$ (c) $|q| = 2\pi/\sqrt{3}$

Fig. 13. Angular dependences of phonon phase velocities.

shown separately for acoustic and optical phonons in Fig. 12.

We also give polar diagrams of phonon phase velocities (Fig. 13). In the long-wave range, phase velocities of both acoustic and optical phonons do not depend on the angle and, thus, the lattice behaves as an isotropic medium (Fig. 13a). With the wavelength decreasing, an angle dependence of the phase velocity begins to manifest itself (Figs. 13b, 13c), i.e., anisotropic properties of the medium show up.

CONCLUSIONS

The performed analysis of dispersion properties of two-dimensional periodic structures showed that, with respect to long-wave perturbations belonging to both acoustic and optical dispersion branches, phonon crystals behave as isotropic media. At wavelengths comparable with the lattice period, acoustic anisotropy begins to manifest itself and there appear preferential directions along which the phase velocity of phonons substantially exceeds the velocities of wave propagation along other directions. Frequency domains where phase and group velocities can be oppositely directed are formed. In complex lattices containing two particles in the Brave cell, there appear opacity regions between acoustic and optical types of oscillations. A possibility of controlling dispersion properties of a phonon crystal by varying microstructure parameters is demonstrated.

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