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ACOUSTIC SIGNAL PROCESSING AND COMPUTER SIMULATION

Ultrasonic Guided Wave Focusing by a Generalized Phased Array[1]

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Abstract—Ultrasonic guided wave focusing by a generalized phased array is studied based on dispersion curves in a multi-layered medium. The different phase of the guided waves with different frequency is added on the excitation signal on each element of the transducer array for focusing. This can be realized by designing a proper excitation pulse based on the dispersion curves of the guided waves for each of the transducer array elements. The numerical simulation results show that the guided waves with different modes, different frequency components, and from different elements of the transducer array can all be focused at the target and focusing is achieved.

Keywords: guided wave, focusing in layered medium, phased array

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1. INTRODUCTION

Ultrasonic guided waves exist in layered media and propagate along the media interface. An important property of guided waves is dispersive and guided waves have multiple modes [1–4]. Guided waves with different frequency component propagate with different velocity, and thus the travel time of the different frequency component of guided waves from a source to a receiver is different.

Different mode of guided waves has different dispersion property. At one single frequency, guided waves have multiple modes propagating with different velocities. If a source signal contains a broad frequency band, all modes of guided waves at all frequencies will arrive at a receiver asynchronously, which results in defocusing.

How to make the dispersive guided waves being focused? Previous work mainly uses acoustic time reversal to achieve focusing for guided waves [5, 6]. It is difficult to artificially control beam focusing by using the acoustic time reversal method. However, an ultrasonic phased array can scan a testing area easily. There are many research activities on the ultrasonic phased array and this method has been applied in many industrial areas successfully [7–9]. The current ultrasonic phased array technique is only based on body waves in homogenous medium. However, to date, it does not appear research on guided waves using ultrasonic phased array method. In this paper, a novel focusing method for highly dispersive guide waves is proposed. The basic principle of this method is to design an excited source signal based on the dispersion property of guide waves for each array element so that the guided waves generated by the designed source signals from all transducer array elements will arrived at a target simultaneously. The key of this method is to control the phase for each frequency component of the transmitted signal, that is, to transmit the slower components of the guided waves earlier than the faster components so that the guided waves with all frequency components will arrive at the target at the same time to achieve focusing. This focusing method can be considered as a generalized phased array focusing method.

2. FOCUSING METHOD

Guided waves exist in stratified media and the most common layered media is horizontal layered. Let us consider a planar layered half-space solid media with a free surface. A transducer array with N elements is placed on the free surface and each element generates acoustic signal by electric excitation. In this paper, only guided waves are considered, i.e., it is assumed that the acoustic waves excited by the transducer are only guided waves. Based on wave propagation theory in multilayered media, the propagation velocity of the guided waves at each frequency can be obtained by solving dispersion equation. The algorithm for finding dispersion curves and displacement components of the guided waves can be found in references [1–4] and shall not be discussed here. For convenience, the free

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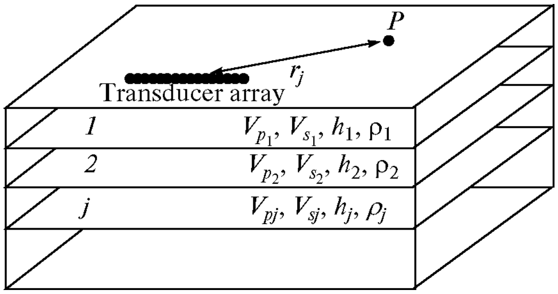

Fig. 1. Configuration of the layered half-space and the transducer array.

surface is taken as a horizontal plane, and it is also assumed that each transducer element has no size dimension and is excited by a vertical directional force. Consider an arbitrary point P in the media, the horizontal distance from P to the jth element of the transducer is r_j ($j = 1, 2, ..., N$). Figure 1 displays the configuration of the layered half-space and the transducer array. If the distance r_j is far greater than the longest wavelength of the guided waves, the displacement of the guided waves for mode m in the frequency domain at point P excited by the jth element can be written as

$$u_{jm}(r_j, \omega) = A_{jm}e^{ik_m r_j}, \tag{1}$$

where $k_m = \omega/v_m$, ω is angular frequency, v_m is the phase velocity of mode m, and A_{jm} is the amplitude which is related to the parameters in each layer and the vertical distance between the transducer array and point P. The time response of the displacement at P then is expressed as

$$u_{jm}(r_j, t) = \int_{-\infty}^{\infty} A_{jm}e^{i\omega(r_j/v_m - t)}F(\omega)d\omega, \tag{2}$$

where $F(\omega)$ is the source spectrum. Equation (2) shows that the guided waves with different frequency component have different propagation velocities and therefore have different phases. Now let us design a new excitation pulse and the frequency spectrum of the new pulse is expressed as

$$F'_{jm}(\omega) = e^{i\omega(r_j/v_0 - r_j/v_m)}F(\omega), \tag{3}$$

where v_0 is an introduced reference velocity, and it is arbitrary and can be the average velocity of the guided waves or the maximum or minimum velocity of the guided waves. The time response of the guided waves with the new excitation pulse is written as

$$\begin{aligned} u_{jm}(r_j, t) &= \int_{-\infty}^{\infty} A_{jm}e^{i\omega r_j/v_m}F'_{jm}(\omega)e^{-i\omega t}d\omega \\ &= \int_{-\infty}^{\infty} A_{jm}e^{i\omega(r_j/v_0 - t)}F(\omega)d\omega. \end{aligned} \tag{4}$$

With the new excitation pulse $F'_{jm}(\omega)$, the guided waves with all frequency components will arrive at point P simultaneously and focusing is achieved. The guided waves generated by the excitation pulse with the traditional frequency spectrum is not focused at point P, while the guided waves can be focused at point P when the source signal on each transducer element is excited by the new pulse spectrum $F'_{jm}(\omega)$. The key for implementing this method is to introduce and design a new excitation pulse called focused pulse with the spectrum $F'_{jm}(\omega)$ which is related to both element location r_j and mode number m.

If there are M guided wave modes within a given frequency range, the displacement of the guided waves at point P excited by the jth transducer element can be expressed as

$$u_j(r_j, t) = \int_{-\infty}^{\infty} \sum_{m=1}^{M} A_{jm}e^{i\omega(r_j/v_m - t)}F'_j(\omega)d\omega. \tag{5}$$

Then the spectrum of the focused pulse will be

$$F'_j(\omega) = e^{i\omega r_j/v_0}F(\omega)\sum_{m=1}^{M} e^{-i\omega r_j/v_m} = \sum_{m=1}^{M} F'_{jm}(\omega), \tag{6}$$

and Eq. (5) now becomes

$$u_j(r_j, t) = \int_{-\infty}^{\infty}\left(\sum_{m=1}^{M} A_{jm} + \sum_{m=1}^{M} A_{jm}\sum_{\substack{n=1 \\ n \neq m}}^{M} e^{i\omega(r_j/v_m - r_j/v_n)}\right) \times e^{i\omega(r_j/v_0 - t)}F(\omega)d\omega. \tag{7}$$

Equation (7) indicates that different modes and different frequency components of the guide waves arrive at point P simultaneously and focusing is achieved. The first term in the right hand side of Eq. (7) is the main lobe of the received guided waves and the second term is the interference among different modes, which distributes on the both sides of the main lobe.

The displacement components in the equations above can be in arbitrary direction. For convenience, we study and analyze the vertical displacement component in the following numerical simulation and analyses.

For N-element transducer array, the focused pulse of each transducer element is different for a given focal point (see Eq. (6)). If all elements are excited by their focused pulses, the guided waves from each element will arrive at the focal point simultaneously because the effect of distance between adjacent elements is eliminated by the focused pulses (Eqs. (6) and (7)).

Unlike the conventional phased array method, the generalized phased array method is to control the excitation pulse in time domain on each transducer element. The principle of the new method is that the designed focused pulse makes the slower components of the guided waves transmitted earlier than the faster components so that all modes with their frequency components arrive at the focal point simultaneously and focusing is achieved. When the excitation pulse for each element is changed gradually as the target moving point-to-point in the detection area, the whole detection area is scanned by the ultrasonic phased array with the guided waves.

3. NUMERICAL SIMULATION AND ANALYSIS

The new idea is validated by numerical simulations. Only the vertical component of the displacement of the guided waves is used for focusing study and analysis. Set a target P on the free surface. For a stratified half-space, it is known that the amplitude of the first mode of the guided waves is much larger than that of other modes when the shear wave velocity increases monotonically as the layer ordinal increases downwards [3, 4]. In this case, the first mode is dominant and the influence of other modes can be neglected. It is also known that the higher-order modes are dominant in the high frequency range in the stratified half-space with low-velocity inserted layers [3, 4]. In this situation both the first mode and higher-order modes should be included. Consider a four-layered model with a low-velocity interlayer and the model parameters are listed in table. Figure 2 displays the phase velocity and displacement as a function of frequencies of the guided wave modes. It shows that there are 11 dispersion modes excited in the frequency range of 0 to 300 kHz (Fig. 2a). Figure 2b shows the displacement for each mode excited by the *j*th transducer element. The first mode dominant in the low frequency range and the next six modes ($m = 2, ..., 7$) have the relatively large amplitude (roughly one third of the amplitude of the first mode) when frequency increases, while the rest of the modes are barely visible.

Model parameters of a four-layered media

Note	Shear-wave velocity, m/s	Compressional-wave velocity, m/s	Density, kg/m^3	Thickness, m
1	3040	6420	2700	0.03
2	1100	2680	1800	0.01
3	2500	4000	2500	0.01
4	3251	5940	7800	∞

To calculate the focused pulse and waveform, the reference velocity v_0 is chosen as the phase velocity of the first mode at the high frequency limit, which is equal to 1124 m/s. Figure 3a shows the traditional cosine envelope excitation pulse of the spectrum $F(\omega)$ in the frequency range of 0 to 300 kHz. The transducer element is excited by a vertical point force. Figure 3b displays the focused pulse of the spectrum $F'_{jm}(\omega)$ when only the first mode is used for focusing. The distance r_j from the *j*th transducer element to target P is 1*m*. It is apparent that the traditional cosine envelope of the spectrum $F(\omega)$ is relatively narrow, while the focused pulse of the spectrum $F'_{jm}(\omega)$ is greatly elongated and its waveform is irregular. Figures 3c and 3d illustrate the received signal at target P when the *j*th element is excited by the cosine envelop with

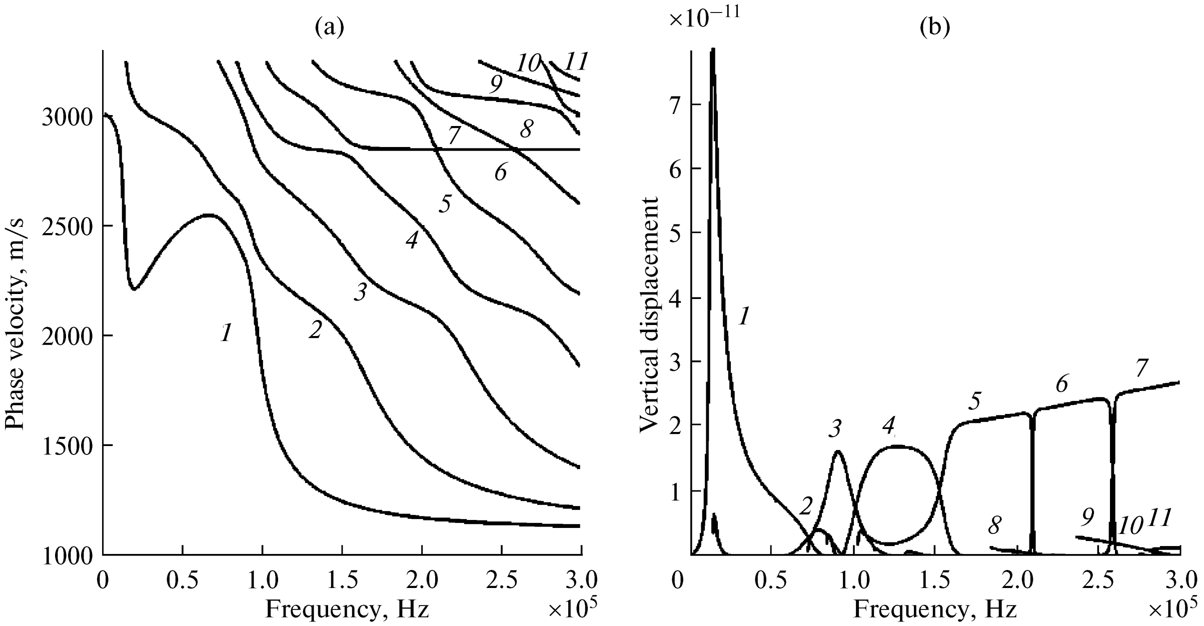

Fig. 2. The phase velocity (a) and vertical component of the displacement (b) of the guided waves excited by a vertical force on the free surface.

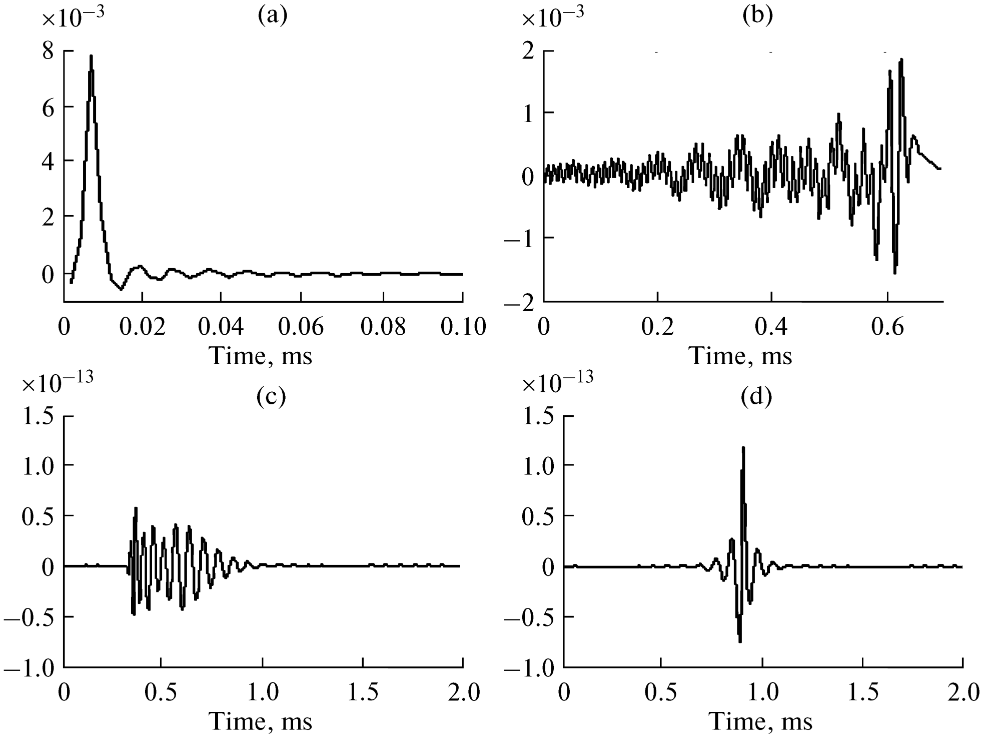

Fig. 3. The excitation signals of the jth element and received waveforms at the target when only the first mode is considered in the frequency range of 0–300 kHz. (a) The cosine envelope excitation pulse, (b) the focused pulse, (c, d) the waveforms at the target when the jth element is excited by the cosine envelope and the focused pulse, respectively.

spectrum $F(\omega)$ and the focused pulse of the spectrum $F'_{jm}(\omega)$, respectively. Figure 3c shows a strong dispersive effect and the waveform is gradually elongated in propagation course because the guided wave velocity is different at different frequency. However, the waveform in Fig. 3(d) is condensed in a narrow range that is because the focused pulse of the spectrum $F'_{jm}(\omega)$ makes all the frequency components of the guided waves arrived at the target simultaneously. The maximum amplitude in Fig. 3d is about 1.3 times of that in Fig. 3c. It is shown that the guided waves with different propagation velocities can be focused satisfactorily by using the focused pulse on the transducer elements.

In the second example, the focusing of the guided waves with more than one mode is studied, while other conditions keep unchanged. All eleven guided wave modes shown in Fig. 2 are considered to achieve focusing using Eqs. (5)–(7). In this case the focused pulse spectrum is calculated by Eq. (6) and the focused pulse is displayed in Fig. 4a, which is little similar to that in Fig. 3b but more complicated. Figures 4b and 4c plot the waveforms at target P when transducer element j is excited by the cosine envelope and the focused pulse, respectively. The waveforms are more complicated since they contain eleven highly dispersive modes in the considered frequency range. The waveform in Fig. 4c is relatively narrower and the amplitude of the waveform is greater than that in Fig. 4(b). The maximum amplitude in Fig. 4c is about 3 times of that shown in Fig. 4b. With the focused pulse excitation, all modes with different frequency components arrive at target P simultaneously and are focused at the target. It can be seen from Fig. 4c that the waveform is little more pell-mell that results from the interference among the different modes. This property can also be seen from Eq. (7) where the second term is the interference among the modes. Figure 4c also shows that the interference does not affect the maximum amplitude and distributes on both sides of the maximum amplitude because the propagation velocity of the interference is different from that of the maximum amplitude.

Finally, a linear ultrasonic transducer array with 16 elements is studied to achieve guided wave focusing. The focused pulse spectrum given by Eq. (6) for each element is different as the distance from each element to target P is different. Assume that the distance between the centers of two adjacent elements is $0.01m$. Figure 5 shows the waveforms at target P which is on the extension line of the transducer array and the distance from target P to the nearest element of the array is $1m$. The frequency range is from 0 to 300 kHz and there exists eleven guided wave modes. In Fig. 5a, the excitation pulse for each element is still cosine envelope. To eliminate the effect of distance among the elements of the transducer array, different element is excited with a different time delay. The time delay

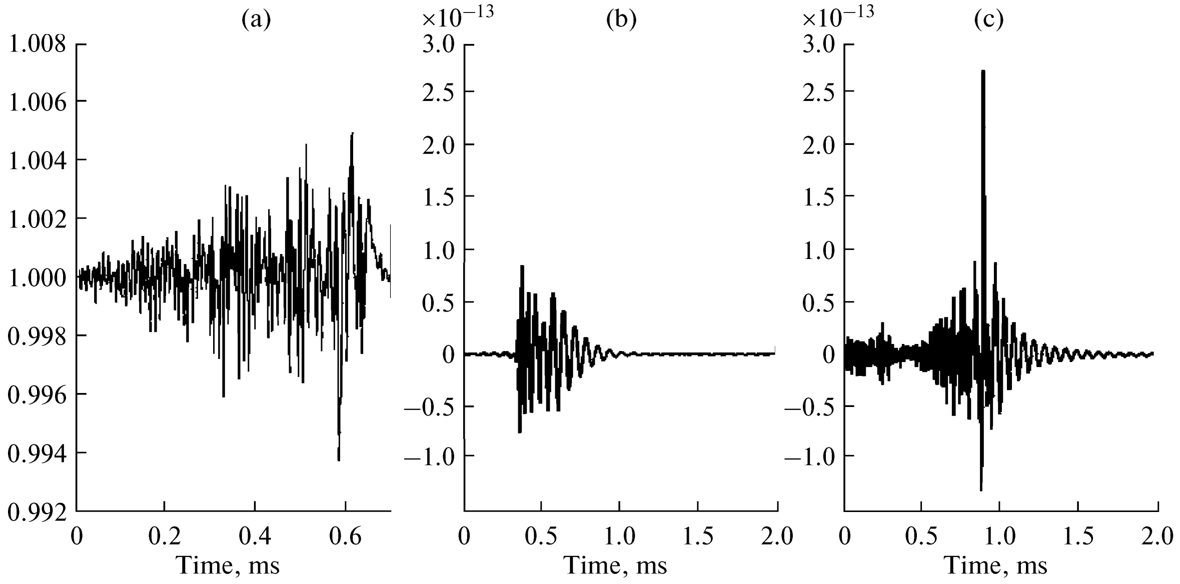

Fig. 4. The excitation signal of the *j*th element and the received waveforms at the target when all of 11 modes of the guided waves are considered: (a) the focused pulse, (b, c) the waveforms at the target when the *j*th element is excited by the cosine envelope pulse and the focused pulse, respectively.

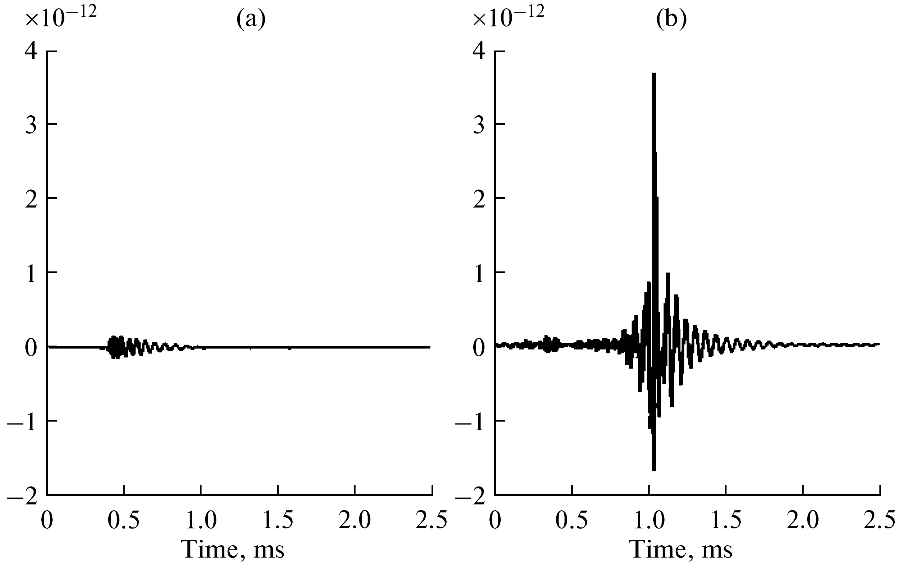

Fig. 5. The received waveforms at target *P*, when the transducer elements excited by the cosine envelope (a) and the focused pulse (b), respectively.

between the excitation pulses of two adjacent elements is determined by the distance between two elements divided by the reference velocity v_0. It shows that the amplitude of the waveform does not increase significantly by using the 16-element array and the maximum amplitude is almost the same as that excited by using only one element. This is because the frequency components of the guided wave with velocity v_0 will arrive at target P simultaneously, while the frequency components of the guided wave with other velocities will arrive at target P asynchronously. Figure 5b is the received waveform at target P when all the elements of the transducer array are excited by their focused pulses. It is found that the amplitude of the waveform increases significantly compared to that in Fig. 5a. The guided waves from different transducer elements and with different modes and different frequency components are all focused at target P. It can be concluded that the amplitude of the waveform increases as the number of the transducer elements increases when the proposed focusing method is adopted.

4. CONCLUSION AND DISCUSSION

In this paper, a novel focusing method for highly dispersive guided waves is proposed. The method utilizes the generalized ultrasonic phased array to give different phase for different frequency component of the guided waves. The slower guided wave is transmitted earlier, while the faster guided wave is transmitted later so that all modes and all frequency components of the guided waves arrive at the focal point simultaneously and focusing is realized. The realization of this method is to design a new excitation pulse (focused pulse) in time domain for each array element to achieve focusing for broadband guided waves. As the conventionally ultrasonic phased array method, the generalized phased array method can achieve ultrasonic guided wave focusing and scan the whole detection area. However, the difference from the conventional method is that the generalized method controls the excitation pulse, while the conventional method controls time delay. The focused pulse is determined by the dispersion curves of the guided wave and the distances between the array elements and the focal point. Therefore, it is easy to conclude that the focusing and scanning of the ultrasonic phased array with the guided waves could be realized if the excitation pulse for each element is changed gradually as the focal point scans the detection area. In reality it is not difficult to add the focused pulse to each array element and excite acoustic signal using current D/A transmitting technique. Time reversal method is to reverse the received complex waveform, add to transducer elements and transmit again, which has been widely proofed by experiments. Similarly, it is feasible to use the D/A technique to generate the focused pulse on each transducer element for the generalized phased array.

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