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PHYSICAL ACOUSTICS

An Arbitrary-Shaped Acoustic Cloak with Merits Beyond the Internal and External Cloaks[1]

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Abstract—Based on transformation acoustics, an arbitrary-shaped acoustic cloak capable of functioning as an information exchange-enabling internal cloak and a movement-allowing external cloak is presented. The general expressions of material parameters for the acoustic cloaks with arbitrarily conformal or non-conformal boundaries are derived, and then the performances of developed cloaks are validated by full-wave simulations. Finally, the different characteristics of the linear and nonlinear transformations-based cloaks are compared and analyzed. The proposed cloak could lead to wider applications beyond that of normal cloaks, since it effectively compensates the insufficiencies of traditional internal and external cloaks. Besides, this work also provides a new method to design bifunctional device and suggests an alternative way to make a large object invisible.

Keywords: metamaterials, transformation acoustics, invisible cloak, finite element method

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1. INTRODUCTION

Transformation acoustics, as one of the major extended branches of transformation optics [1–9], has recently received considerable attention in the scientific community. Based on the form-invariance of acoustic wave equations under the coordination transformation, transformation acoustics has paved a new way for the arbitrary control of acoustic wave using metamaterials. It has also shaped up a revolutionary design paradigm that enables researchers to create astonishing acoustic devices previously deemed impossible or unconceivable, including invisible cloak [10–18], superscatterer [19], planar hyperlens [20], concentrator [21], illusion device [22], and rotator [23]. It is undoubtedly true that the acoustic invisible cloak is the most intriguing one among them, and potential applications of cloak bear substantial significance in both military and civilian uses, such as hiding submarines from enemy's active sonar, protecting a particularly sensitive section of a structure against blast or shock waves, acoustic noise reduction by creating sound-shielding materials, and seismic isolation of civil infrastructure [24]. Generally speaking, two main kinds of acoustic cloaks are available: internal cloak [10–15] and external cloak [16–18]. The internal cloak can make whatever it covers undetectable by steering acoustic wave around an enclosed domain, yet the object concealed by such kind of cloak does not communicate with the outside world since no wave can reach into the hidden area, and vice-versa. By employing an anti-object to acoustically cancel the scatterings of hidden object, the external cloak leaves the hidden object out in the open, so that it can exchange information with surroundings. But for this cloak, there exists a strict one-to-one correspondence between positions of anti-object and hidden object, and thus once the position of anti-object is fixed, the hidden object cannot move. Recently, electromagnetic internal-external cloak which combines the use of above-mentioned two cloaks has been reported [25, 26], and the results could also be easily extended from electromagnetics to acoustics through a simple variable substitution [11]. However, it should be pointed out that this kind of cloak still has drawbacks of internal and external cloaks themselves.

To remove the bottlenecks of internal and external cloaks, we propose a more feasible and practical acoustic cloak with arbitrary shapes based on transfor-

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mation acoustics. Firstly, it not only supports the function of traditional internal cloak, but also enables the hidden object to exchange information with the outside world. Secondly, it inherits the virtues of traditional external cloak, as well as allows the hidden object to move within the cloak. Full wave simulations by the finite element method are performed to confirm the performance of suggested cloak. Moreover, the effect of linear and nonlinear transformation equations on the characteristics of the acoustic cloak is investigated. This work expands the application of transformation acoustics and greatly improves the design flexibility of cloak.

2. THEORY AND SIMULATION MODEL

The derivation of material parameters for an arbitrary-shaped acoustic cloak is based on transformation acoustics, the fundamental of which lies in that acoustic wave equation is form-invariant under coordinate transformations. Without a source term, acoustic wave equation in the virtual space, which represents the original space before performing coordinate transformation, is given by

$$\nabla\left[\frac{1}{\rho}\nabla p\right] = -\frac{\omega^2}{\kappa}p, \tag{1}$$

where ρ, p, ω and κ are the mass density, the acoustic pressure, the angular frequency and the bulk modulus, respectively. ∇ represents the gradient operator. As has been proved by Chen *et al.* [10] that the acoustic wave equation retains its form under the coordinate transformation of $x' = x'(x)$, Eq. (1) in the physical space can be written as

$$\nabla'\left[\frac{1}{\rho'}\nabla p'\right] = -\frac{\omega^2}{\kappa'}p'. \tag{2}$$

Note that Eqs. (1) and (2) have the same structure, except that the material parameters (*i.e.*, ρ' and κ') in the physical space should satisfy the following form:

$$\rho'^{-1} = \begin{bmatrix} \rho'^{-1}_{xx} & \rho'^{-1}_{xy} & \rho'^{-1}_{xz} \\ \rho'^{-1}_{yx} & \rho'^{-1}_{yy} & \rho'^{-1}_{yz} \\ \rho'^{-1}_{zx} & \rho'^{-1}_{zy} & \rho'^{-1}_{zz} \end{bmatrix} = \rho^{-1}\left[\frac{JJ^T}{\det J}\right],\ \kappa' = \kappa\det J, \tag{3}$$

where $J = \dfrac{\partial(x',y',z')}{\partial(x,y,z)} = \begin{bmatrix} \partial x'/\partial x & \partial x'/\partial y & \partial x'/\partial z \\ \partial y'/\partial x & \partial y'/\partial y & \partial y'/\partial z \\ \partial z'/\partial x & \partial z'/\partial y & \partial z'/\partial z \end{bmatrix}$ denotes the Jacobian transformation matrix between the virtual space and the physical space. J^T and $\det J$ are the transpose and the determinant of matrix J, respectively. For simplicity we consider the case of 2D arbitrary-shaped acoustic cloak under plane wave irradiation, for which only ρ'^{-1}_{xx}, ρ'^{-1}_{xy}, ρ'^{-1}_{yx}, ρ'^{-1}_{yy} and κ' components of material parameters in Eq. (3) are relevant. In what follows, we are concerned with the question of how to derive the material parameters of 2D arbitrary-shaped acoustic cloak based on aforementioned theory.

Figure 1 shows a schematic diagram of the proposed 2D arbitrary-shaped acoustic cloak, in which three concentric cylinders with contours of $R_1(\theta)$, $R_2(\theta)$ and $R_3(\theta)$ divide the physical space into three regions, i.e., hidden area [region I, $0 \le r' \le R_1(\theta)$], complementary media layer [region II, $R_1(\theta) \le r' \le R_2(\theta)$] and outer area [region III, $R_2(\theta) \le r' \le R_3(\theta)$]. It should be mentioned out that the objects that need to be hidden are placed inside the region I. To achieve the extraordinary features of acoustic cloak, we fold the region $0 \le r \le R_1(\theta)$ in the virtual space into the region $R_1(\theta) \le r' \le R_2(\theta)$ in the physical space while keeping the outer boundary $r' = r = R_3(\theta)$ invariant. Here, r and r' represent the radius of the virtual space and the physical space, respectively. The mapping procedures outlined above lead to the following boundary conditions: $R_1(\theta) = r'_{\mathrm{II}}(R_1(\theta))$, $R_2(\theta) = r'_{\mathrm{II}}(0)$ for regions II and $R_3(\theta) = r'_{\mathrm{III}}(R_3(\theta))$, $R_2(\theta) = r'_{\mathrm{III}}(0)$ for region III. Then, based on these boundary conditions, linear transformation equations for the regions II and III between the virtual and physical spaces can be summarized as

$$r' = r'_{\mathrm{II}}(r) = \frac{R_1(\theta) - R_2(\theta)}{R_1(\theta)}r + R_2(\theta), \quad \theta' = \theta, \quad z' = z, \tag{4}$$

$$r' = r'_{\mathrm{III}}(r) = \frac{R_3(\theta) - R_2(\theta)}{R_3(\theta)}r + R_2(\theta), \quad \theta' = \theta, \quad z' = z. \tag{5}$$

In Cartesian coordinates, the above equations can be further written as

$$x' = \frac{R_1(\theta) - R_2(\theta)}{R_1(\theta)}x + R_2(\theta)\frac{x}{\sqrt{x^2+y^2}}, \tag{6a}$$

$$y' = \frac{R_1(\theta) - R_2(\theta)}{R_1(\theta)}y + R_2(\theta)\frac{y}{\sqrt{x^2+y^2}}, \tag{6b}$$

$$z' = z \tag{6c}$$

for the region II and

$$x' = \frac{R_3(\theta) - R_2(\theta)}{R_3(\theta)}x + R_2(\theta)\frac{x}{\sqrt{x^2+y^2}}, \tag{7a}$$