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Modulating Lamb Wave band Gaps using an Elastic Metamaterial Plate[1]

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Abstract—Modulating band gaps (extending the bandwidths or shifting into a lower frequency range) is a challenging task in phononic crystals. In this paper, elastic metamaterial plates composed of a square array of "hard" stubs or "soft" stubs on both sides of a 2D binary locally resonant plate are proposed, and their band structures are studied. The dispersion relationships and the displacement fields of the eigenmodes are calculated using finite element methods. Numerical results show that the band gaps are shifted to lower frequencies and the bandwidths are enlarged compared to classic elastic metamaterial plates. A conceptual "analogous-rigid mode" that includes an "out-of-plane analogous-rigid mode" and an "in-plane analogous-rigid mode" is developed to explain these phenomena. The "out-of-plane analogous-rigid mode" mainly adjusts the band gaps into the lower frequency range, and the "in-plane analogous-rigid mode" mainly enlarges the bandwidth. Furthermore, the band gap effects of composite "hard" stubs and "soft" stubs are investigated. The results show that the location of the band gaps can be modulated into a relatively lower frequency and the bandwidth can be extended by the use of different composite stubs. These elastic wave properties in the proposed structure can be used to optimize band gaps and possibly produce low-frequency filters and waveguides.

Keywords: elastic metamaterial plate, out-of-plane analogous-rigid mode, in-plane analogous-rigid mode, modulating Lamb wave band gaps

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1. INTRODUCTION

In the last two decades the propagation of acoustic and elastic waves in periodic composite materials called acoustic/elastic metamaterials or phononic crystals (PCs) have received much attention. For example, phononic band gaps (BGs) in which acoustic wave propagation is forbidden have been shown to exist [1–4]. There are two mechanisms that form BGs: Bragg scattering and localized resonance. Band gaps are generated by Bragg scattering and their wavelengths are of the same order as the period. The width and position of Bragg band gaps depend heavily on the contrast between the elastic parameters of the scattering and the host material, as well as the geometric parameters and the shape of the inclusions [5–10]. A large lattice constant is needed to obtain low frequency range band gaps, which hinders the application of PCs in the audible frequency range. The associated band gap wavelengths generated based on localized resonance (first proposed by Liu et al. [11]) are two orders of magnitude smaller than Bragg band gaps. A resonant band gap relies on the resonance frequency of the scattering units and depends less on the periodicity and symmetry of the structure. This addresses the limitations of Bragg band gaps, and thus presents a method of producing low-frequency band gaps. There are many researches on the effect factors and formation mechanisms of locally resonant BGs. However, little attention has been focused on the study on how to expand the BGs in the low-frequency range, although the BGs are very narrow.

More recently, the propagation of Lamb waves (elastic waves) in elastic metamaterials plates (also called as PC-plates) has attracted much attention due to their potential applications in filters, resonators, waveguides, and vibration insulation [12–33]. In general, elastic metamaterials plates can be classified into two types according to their structural features: flat plates and stubbed plates. A flat plate consists of a periodic array of holes/inclusions in a homogeneous plate. Many studies have previously focused on the band gaps behaviors of this type of elastic metamaterials plate [12–15]. However, it is not easy to obtain lower-frequency BGs in it. Wu et al. [15] studied wave

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propagation in an elastic metamaterials plate composed of a square array of cylindrical stubs on one side of a homogeneous plate and demonstrated that band gaps can be obtained at low frequencies when the stub height is roughly three times the plate thickness. Much research has been conducted to extend this elastic metamaterials plate, resulting in stubbed or pillared elastic metamaterials plates [17–30]. The one-sided stubbed elastic metamaterials plate is composed of a square array of stubs on one side of a homogeneous plate. Pennec et al. [17] investigated the effects of material properties on the BGs of one-sided stubbed elastic metamaterials plates composed of a square array of two-layer stubs on one side of a homogeneous plate. Oudich et al. [18] studied the band structure of a one-sided stubbed elastic metamaterials plate composed of a square array of three-layer stubs on a homogeneous plate. Zhang et al. [20] studied the effects of geometric parameters on the BGs of one-sided stubbed elastic metamaterials plate with tapered stubs; they obtained lower BGs in comparison to a similar elastic metamaterials plate composed of cylindrical stubs. Hsu et al. [21] studied the BGs of one-sided stubbed elastic metamaterials plate with neck stubs and demonstrated that both localized resonance band gaps and Bragg band gaps occur simultaneously. Bilalet et al. [22] studied the band structure of a stubbed elastic metamaterials plate consisting of a periodic array of holes in an one-side stubbed elastic metamaterials plate and indicated that the localized resonant band gaps increased by a factor up to 4 due to the trampoline effect. Assouar et al. [23] proposed a stubbed elastic metamaterials plate consisting of a square array of stubs on both sides of a homogenous plate; a significant bandwidth expansion—a factor of 2 compared to a one-side stubbed elastic metamaterials plate – was obtained.

Nevertheless, all these stubbed elastic metamaterials plates were constructed by depositing one-sided or double-sided stubs on a homogeneous plate. Li et al. [33] proposed a one-sided stubbed elastic metamaterials plate composed of a square array of simple stubs on one side of a 2D binary local resonant PC-plate and studied its Lamb wave propagation properties. Compared to the classic one-sided stubbed elastic metamaterials plate, the band gap shifts more readily to the lower frequency range and the bandwidth is expanded.

The BGs of these elastic metamaterials plates are usually located in the frequency range above 300 Hz, and the bandwidths are narrow. However, most practical applications require band gaps with a wide bandwidth or low frequency range from 20 to 250 Hz. Thus, modulating band gaps (extending the bandwidths or shifting into a lower frequency range) is still a challenging task.

In this research we studied the BGs of Lamb waves in an elastic metamaterials plate (a novel PC-plate) composed of a square array of "soft" or "hard" stubs on both sides of a 2D binary locally resonant PC-plate. The dispersion relationships and the displacement fields of the eigenmodes were calculated using finite element methods (FEM). Results show that bandwidths are expanded and band gaps are shifted to lower frequencies in comparison to classic double-sided stubbed elastic metamaterials plates. The forming mechanism of modulating BGs and the effects of composite stubs on BGs were investigated. As a result, a new "analogous rigid mode" including an "out-of-plane analogous-rigid mode" and an "in-plane analogous-rigid mode" of the stub is presented.

2. MODEL AND METHOD OF CALCULATION

The proposed structures consist of a square array of composite stubs on both sides of a 2D binary locally resonant plate. The 2D binary locally resonant plate is fabricated by filling the drilled holes of a finite thickness epoxy plate with rubber filler. The composite stubs are composed of stub A (made of soft materials) and stub B (made of hard materials). Figure 1 shows different unit cells of the proposed structures. According to the characteristics of composite stubs, we classified the proposed structures into two types. The one is the double-sided stubbed composed elastic metamaterials plates with "hard" stubs (Fig. 1a), which consist of "hard" stub such as steel stubs contact with rubber filler; the other is the double-sided stubbed composed elastic metamaterials plate with "soft" stubs (Fig. 1b) which consist of "soft" stub such as rubber stubs contact with rubber filler. The complete structure is an infinite repetitive arrangement of the unit cells along the x- and the y-directions. The diameter of the rubber filler, the epoxy plate thickness, and the lattice constant are denoted by D, e and a, respectively. The height and diameter of the stub are denoted by h (stub A is denoted by h_R, stub B is denoted by h_S) and d, respectively. The material parameters used in the calculations are shown in Table 1. Materials A and B are rubber and steel, respectively.

In order to investigate band gaps and resonant modes in the proposed structures, a series of dispersion relationships are calculated using FEM based on the Bloch theorem; this has been proven as an efficient method [32, 33]. Commercially available software (Comsol Multiphysics 3.5a) was used to implement the FEM calculation. Since the infinite system is periodic along the x- and y- directions simultaneously, only the single unit cell shown in Fig. 1 needs to be considered. The unit cell is meshed by using a triangular mesh with the Lagrange quadratic elements provided by Comsol. Good convergence will be obtained