ISSN 1063-7710, Acoustical Physics, 2017, Vol. 63, No. 5, pp. 617–624. © Pleiades Publishing, Ltd., 2017.

PHYSICAL FUNDAMENTALS
OF ENGINEERING ACOUSTICS

Mode Analysis of Trilaminar Bender Bar Transducers using an Approximation Method[1]

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Received July 15, 2016

Abstract— Based on the vibration theory of a thin plate, an analytical treatment of the trilaminar bender bar with piezoelectric elements and inert substrate of various lengths is presented for mode analysis. Resonance frequency and effective electromechanical coupling coefficient are calculated by this method. The impacts of the geometries of the bender bar on the performance of its fundamental and third-order flexural mode are investigated in detail under rigid boundary conditions. It is shown that resonance frequency is extremely sensitive to the thickness of inert substrate. Moreover, the effective electromechanical coupling coefficient has peaks as the length of piezoelectric elements varies. The peaks are achieved when the length of piezoelectric elements equals the length between two nodes having zero strains in the *x*-direction. The trilaminar bender bar will be effectively excited when the strains on the piezoelectric element are in the same phase, which is important to disclose the vibration mechanisms of this kind of transducer. Also, analytical results are compared with the ones of numerical simulation. The results suggest that effective electromechanical coupling coefficient shares similar patterns with electrical conductance, which can be used to characterize transducer performance to a certain extent. It also demonstrates that the analytical treatment provides an efficient alternative way for optimizing the bender bar transducer design.

Keywords: trilaminar bender bar, approximation theoretical analysis, resonance frequency, effective electromechanical coupling coefficient

DOI: 10.1134/S1063771017050049

INTRODUCTION

Transducers utilizing flexural-mode vibrations are more compact in size than those operating in extensional modes. Compactness is achieved at the cost of a reduction in power capability [1, 2]. At low frequencies, extensional vibrators usually tend to be very large in size, and the power will be higher than needed. Thus, flexural mode transducers are suitable to application for low frequency excitation where large size transducers would be impractical [3, 4]. Typically, the trilaminar bender bars operating in flexural modes are extensively used as low frequency transducers in underwater acoustics and geophysical logging, and their size and weight are reduced to a minimum in consistency with required power.

As for trilaminar bender bars, in which piezoelectric elements equal the inert substrate in length, analytical treatment, such as equivalent circuit method, can be easily used to investigate their performance [5]. Woollett analyzed the bender bar transducer and developed an equivalent circuit model for this bender bar [6]. In many cases, equivalent circuit representations for the fundamental mode may be reduced to the simple Van Dyke form [4]. Similarly, Luan analyzed the two-layer piezoceramic circular plate with the same diameter for both layers [7]. Illustrating with examples of bender bar transducers vibrating in flexural modes, Boris proposed a simple practical way to optimize the effective coupling coefficient by changing the electrode shape [8].

However, the practical situation is that piezoelectric elements are generally shorter than the inert substrate, so the theoretical discussion becomes difficult. Thus, the finite element method (FEM) is widely adopted to simulate the vibrations of bender bars [9–

[1] The article is published in the original.

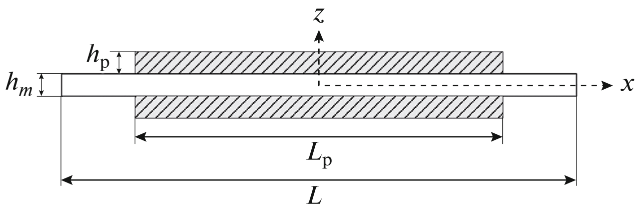

Fig. 1. Geometry of a trilaminar bender bar.

11], but lots of model simulations are much more time-consuming than those done by analytical methods.

Moreover, previous studies concentrated on the fundamental flexural vibration of the bender bar. Extensive usage of the fundamental flexural mode in various applications has given rise to well-developed research on fundamental flexural mode. However, to our knowledge, few effort has been made to examine the higher-order mode behavior of the bender bar, even though such modes may be of significant practical interest as low frequency acoustic transducers in some cases, such as in borehole acoustic wave logging [12, 13].

In our studies, an analytical approach is presented for the trilaminar bender bar transducer with piezoelectric elements shorter than the inert substrate. Resonance frequency and effective electromechanical coupling coefficient of both fundamental and high-order flexural mode can be efficiently obtained by our method, which is essential for the design and optimization of these transducers.

MODEL AND THEORETICAL ANALYSIS

The trilaminar bender bar comprises an inert substrate with a pair of piezoelectric elements attached to two sides in the thickness direction. Generally, the inert substrate is made of inactive metal material to enhance the ability of withstanding hydrostatic stress. The geometry of the trilaminar bender bar is shown in Fig. 1. Suppose z-axis is parallel to the thickness direction, which is also the direction of bending motion. The xy-plane bisects the bar in thickness. The electric field is parallel to the z-axis, the same as the polarization direction of piezoelectric elements.

The operation principle of the bender bar transducer is that when a voltage is applied to the electrical terminals, one of piezoelectric elements experiences extension, while the other experiences contraction, thus resulting in a flexural vibration of the transducer, along with a neutral plane. When the cross-section of the bar is symmetrical about a horizontal plane, this neutral plane will coincide with the central axis of the bar.

In our studies, a thin plate and small deflection theory will be deployed. The total thickness and the width of the trilaminar bender bar will be assumed small compared with its length. The total thickness should be less than 0.16 wavelengths in the plate [14]. As the surfaces of the bar are traction-free, it is assumed that the stress in the volume vanish, except for stress T_{xx}, T_{xy} and T_{yy}, i.e. $T_{xz} = T_{yz} = T_{zz} = 0$. Note that under the assumption of a thin plate, the elementary theory of bending is applicable, which states that the cross-sections remain undistorted and perpendicular to the neutral surface while turning at some angle. The displacement in the z-direction due to turning of the cross-section under bending is denoted as w and is assumed constant over the thickness.

The trilaminar bender bar is divided into two portions in our analysis. The one is a three-layer composite structure with the same length, i.e. the portion with location $|x| < L_p/2$, and the other is a one-layer inert substrate with location $L_p/2 < |x| < L/2$. For the sake of brevity and clarity, in what follows the x-direction is denoted as 1, the y-direction as 2, and the z-direction as 3.

Considering the composite section with $|x| < L_p/2$, we have the following constitutive equations for piezoelectric elements [7]:

$$\begin{cases} T_1 = \dfrac{1}{s_{11}^D\left(1-\sigma_p^2\right)}\left(S_1 + \sigma_p S_2\right) - \dfrac{g_{31}}{s_{11}^D\left(1-\sigma_p\right)} D_3, \\ T_2 = \dfrac{1}{s_{11}^D\left(1-\sigma_p^2\right)}\left(\sigma_p S_1 + S_2\right) - \dfrac{g_{31}}{s_{11}^D\left(1-\sigma_p\right)} D_3, \\ T_6 = \dfrac{1}{2s_{11}^D\left(1+\sigma_p\right)} S_6, \\ E_3 = -\dfrac{g_{31}}{s_{11}^D\left(1-\sigma_p\right)}\left(S_1 + S_2\right) + \overline{\beta}_{33} D_3, \end{cases} \tag{1}$$

where σ_p is the Poisson's ratio of the piezoelectric element at constant electric displacement, and $\sigma_p = s_{12}^D / s_{11}^D$ with s_{11}^D and s_{12}^D being the compliance coefficients; $\overline{\beta}_{33}$ is the permittivity coefficient; g_{31} is the piezoelectric coefficient; E_3 is the electric field component; and D_3 is the component of electric displacement.

The stress components in the inert substrate can be expressed as

$$\begin{cases} T_1' = \dfrac{E_m}{1-\sigma_m^2}\left(S_1 + \sigma_m S_2\right), \\ T_2' = \dfrac{E_m}{1-\sigma_m^2}\left(\sigma_m S_1 + S_2\right), \\ T_6' = \dfrac{E_m}{2\left(1+\sigma_m\right)} S_6, \end{cases} \tag{2}$$

where E_m is the Young's modulus of the inert substrate, and σ_m is its Poisson's ratio.